Equations of motion of the dispersed phase

- Lagrangian approach for the dispersed phase :

$$\frac{d}{dt}\begin{bmatrix} u_P \\ v_P \end{bmatrix} = K_M Re_P \left(C_W(Re_P) \begin{bmatrix} u_F - u_P \\ v_F - v_P \end{bmatrix} + C_M(\sigma) \begin{bmatrix} v_F - v_P \\ -(u_F - u_P) \end{bmatrix} \right)$$

$$+ \frac{\rho_P - \rho_F}{\rho_P + \frac{1}{2}\rho_F} \begin{bmatrix} g_x \\ g_y \end{bmatrix}$$

$$K_M = \frac{3}{4} \frac{\nu\, \rho_F}{(\rho_P + \frac{1}{2}\rho_F)\, d_P^2} \qquad Re_P = \frac{d_P\, v_{rel}}{\nu}$$

$$\sigma = \frac{1}{2} \frac{d_P\, (\Omega - \omega)}{v_{rel}} \qquad v_{rel} = \sqrt{(u_F - u_P)^2 + (v_F - v_P)^2}$$

- $C_W = C_W(Re_P)$ obtained from correlations by Morsi/Alexander
- $C_M = (0.4 \pm 0.1)\, \sigma$ for $|\sigma| \leq 1$; $C_M \equiv (0.4 \pm 0.1)$ for $|\sigma| < 1$
- Source terms in the Navier–Stokes equations due to momentum transfer between phases (PSI–cell model by C.T. Crowe) :

$$S_{u_i}^P = -\frac{1}{V_{ij}} \sum m_P \dot{N}_P \left[u_{Pi,out} - u_{Pi,in} - g_i \left(\frac{\rho_P - \rho_F}{\rho_P + \frac{1}{2}\rho_F} \right) (t_{out} - t_{in}) \right]$$

- **L**agrangian **s**tochastic–**d**eterministic turbulence model (LSD or discret eddy model) as proposed by Sommerfeld, Schönung, Milojević