

## The 3-dimensional eq. of motion for the disperse phase

$$\begin{aligned}
 \frac{d}{dt} \begin{bmatrix} u_P \\ v_P \\ w_P \end{bmatrix} &= \frac{3}{4} \frac{\rho_F}{\rho_P d_P} \left( v_{rel} C_D(Re_P) \begin{bmatrix} u_F - u_P \\ v_F - v_P \\ w_F - w_P \end{bmatrix} \right. \\
 &+ \frac{v_{rel}}{\omega_{rel}} C_M(\sigma) \cdot \begin{bmatrix} (v_F - v_P)(\omega_z - \Omega_z) - (w_F - w_P)(\omega_y - \Omega_y) \\ (w_F - w_P)(\omega_x - \Omega_x) - (u_F - u_P)(\omega_z - \Omega_z) \\ (u_F - u_P)(\omega_y - \Omega_y) - (v_F - v_P)(\omega_x - \Omega_x) \end{bmatrix} \\
 &+ \frac{2\nu_F^{1/2}}{\pi |\vec{\Omega}|^{1/2}} C_A \begin{bmatrix} (v_F - v_P)\Omega_z - (w_F - w_P)\Omega_y \\ (w_F - w_P)\Omega_x - (u_F - u_P)\Omega_z \\ (u_F - u_P)\Omega_y - (v_F - v_P)\Omega_x \end{bmatrix} \\
 &+ \frac{\rho_P - \rho_F}{\rho_P + \rho_F/2} \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} \\
 \\
 \frac{d}{dt} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} &= -\frac{15}{16\pi} \frac{\rho_F}{\rho_P} \omega_{rel} \xi_m(Re_\omega) \begin{bmatrix} \omega_x - \Omega_x \\ \omega_y - \Omega_y \\ \omega_z - \Omega_z \end{bmatrix}
 \end{aligned}$$

with :

$$\vec{\Omega} = \text{rot } \vec{v}_F \quad , \quad Re_P = \frac{d_P v_{rel}}{\nu_F} \quad , \quad Re_\omega = \frac{1}{4} \frac{d_P^2 \omega_{rel}}{\nu_F} \quad , \quad \sigma = \frac{1}{2} \frac{d_P \omega_{rel}}{v_{rel}}$$

$$v_{rel} = \sqrt{(u_F - u_P)^2 + (v_F - v_P)^2 + (w_F - w_P)^2}$$

$$\omega_{rel} = \sqrt{(\omega_x - \Omega_x)^2 + (\omega_y - \Omega_y)^2 + (\omega_z - \Omega_z)^2}$$