# Double Averaged Turbulence Modelling in Eulerian Multi-Phase Flows

Alan Burns[1], Thomas Frank[1], Ian Hamill[1] and Jun-Mei Shi[2].
[1]ANSYS CFX and [2]FZ-Rossendorf

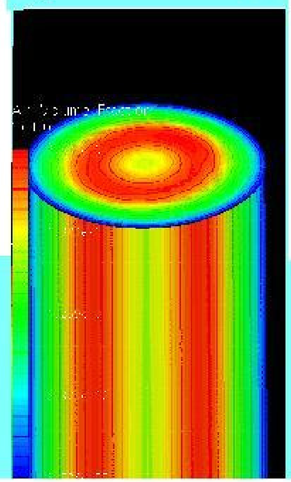
# Physics Of Bubbly Flow

**Momentum Transfer**

Drag, Lift, Wall Lubrication, and Virtual Mass Forces.

Turbulent Dispersion Force.

Responsible for volume fraction distribution.



**Turbulence**

Continuous Phase Affects Dispersed Phase
- Dispersed Phase Reynolds Stresses
- Turbulent Dispersion

Presence of Dispersed Phase Affects Continuous Phase Turbulence - Turbulence Modulation
- Sometimes Enhanced
- Sometimes Reduced.

**Breakup and Coalescence**

Determines Bubble Size Distribution.

Breakup caused by, e.g.
- turbulence
- mean shear
- buoyancy (instabilities)

Coalescence caused by, e.g.
- high bubble volume fractions
- increased probability of collisions.

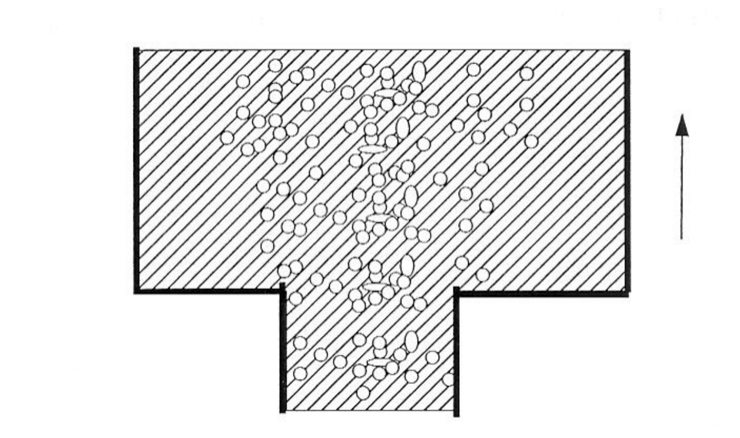
# Multi-Phase Turbulence Phenomena

- **Continuous Phase Effects On Dispersed Phase Turbulence.**
  - **Simplest** model for dilute dispersed phase Reynolds stresses:

$$u'_d = C u'_c \qquad \overline{u'_{id} u'_{jd}} = C^2 \overline{u'_{ic} u'_{jc}}$$

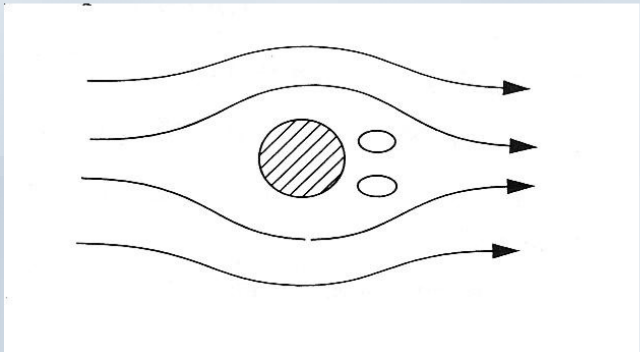
$$\nu_{td} = \frac{\nu_{tc}}{\sigma_{\nu\beta}} \qquad \sigma_{\nu\beta} = \frac{1}{C^2}$$

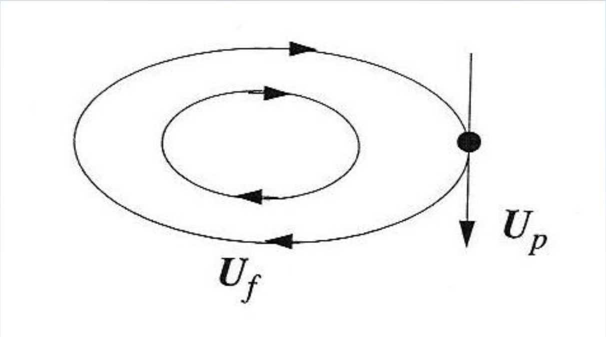
- **Turbulent Dispersion.**
  - Migration of dispersed phase from regions of high to low void fraction.
  - Continuous phase eddies capture dispersed phase particles by action of interfacial forces.



# Multi-Phase Turbulence Phenomena

- **Dispersed Phase Effects On Continuous Phase Turbulence.**
- **Turbulence Enhancement**
  - Due to shear production in wakes behind particles.
- **Turbulence Reduction**
  - Due to transfer of turbulence kinetic energy to dispersed phase kinetic energy by action of interfacial forces.





# Averaging Procedures

- **First Average = Phase Average**
- Phase indicator function:
  - $\chi_\alpha(x,t) = 1$ if phase $\alpha$ is present, $= 0$, otherwise.
- Use ensemble-, time- or space-averaging to define phase-averaged variables:
  - 'Volume Fraction': $r_\alpha = \langle \chi_\alpha \rangle$
  - Material Density $\rho_\alpha = \langle \chi_\alpha \rho \rangle / r_\alpha$
  - Phase Averaged Transport Variable: $\Phi_\alpha = \langle \chi_\alpha \rho \Phi \rangle / \rho_\alpha$
- Essentially **Mass-Weighted Average**

# Averaging Procedures:

- **Phase averaged Momentum and Continuity.**

$$\frac{\partial}{\partial t}(r_\alpha \rho_\alpha U_{\alpha k}) + \frac{\partial}{\partial x_i}\left(r_\alpha \left(\rho_\alpha U_{\alpha i} U_{\alpha k} - \left(\tau_{\alpha ik} + \tau^t_{\alpha ik}\right)\right)\right) = -r_\alpha \frac{\partial P}{\partial x_k} + r_\alpha B_k + M_{\alpha k}$$

$$\frac{\partial}{\partial t}(r_\alpha \rho_\alpha) + \frac{\partial}{\partial x_i}(r_\alpha \rho_\alpha U_{\alpha i}) = 0$$

- $M_{\alpha k}$ = **interfacial forces**
- $\tau^t_{\alpha ik} = -\rho_\alpha \left\langle u'_{i\alpha} u'_{j\alpha} \right\rangle$ **= Reynolds Stress** like terms
- Phase induced turbulence, or full turbulence?
- Some researchers assume this represents full turbulence, e.g. Kashiwa et al.
- We assume it represents **phase-induced turbulence**.

# Averaging Procedures:

- **Models for Phase-Induced Turbulence**.
- **Sato**: Algebraic Eddy Viscosity:

$$\tau_{ij,pi} = -\rho k_{pi}\delta_{ij} + \mu_{t,pi}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial U_i}{\partial x_i}\right) \qquad \mu_{t,pi} = C_\mu \rho_c r_d d_p \left|U_c - U_d\right|$$

- Kataoka and Serizawa (1989) derived exact transport equations for $k_{pi}$ and $\varepsilon_{pi}$.
- Term identified for enhanced turbulence production:

$$P_{k\alpha,pi} = -\vec{M}_{\alpha\beta} \cdot \left(\vec{U}_\beta - \vec{U}_\alpha\right)$$

- Exact once the terms for interfacial forces are closed.

# Averaging Procedures

- **Second Average = Time or Favre Average.**
- Ensemble averaged phase equations are fully space and time dependent.
- Hence, may apply a second *time- average*.
- **Shear induced turbulence**?
- Favre or Mass Weighted averaging is favoured, as it leads to much fewer terms in the averaged equations.

# Favre Averaging

- Favre averaging of phase-averaged variables is defined as follows:

$$\tilde{\Phi}_\alpha = \overline{r_\alpha \rho_\alpha \phi_\alpha} / \overline{r_\alpha \rho_\alpha}$$

- For constant density phases, reduces to a volume fraction weighted average:

$$\tilde{\Phi}_\alpha = \overline{r_\alpha \phi_\alpha} / \overline{r_\alpha}$$

- Favre-averaged and time-averaged quantities are related as follows:

$$\tilde{\Phi}_\alpha = \overline{\Phi_\alpha} + \overline{r'_\alpha \phi'_\alpha} / \overline{r_\alpha}$$

# Favre Averaging

- Time- and Favre- averaged velocities are related by:

$$\tilde{\vec{U}}_\alpha = \overline{\vec{U}}_\alpha + \overline{\vec{u}''_\alpha} \qquad \overline{\vec{u}''_\alpha} = \overline{r'_\alpha \vec{u}'_\alpha} / \overline{r_\alpha}$$

- $\overline{r'_\alpha \vec{u}'_\alpha}$ is fundamental to turbulent dispersion, as it describes how phasic volume fractions are spread out by velocity fluctuations.
- Eddy-viscosity type turbulence models, employ eddy diffusivity hypothesis (EDH):

$$\overline{r'_\alpha \vec{u}'_\alpha} = -\frac{\nu_{t\alpha}}{\sigma_{r\alpha}} \nabla \overline{r_\alpha}$$

- Turbulent Prandtl number is typically of order unity.

# Favre Averaging

- **Time Averaged Continuity Equation**

$$\frac{\partial}{\partial t}\left(\rho_\alpha \overline{r_\alpha}\right)+\frac{\partial}{\partial x_i}\left(\rho_\alpha\left(\overline{U_{\alpha i}}\,\overline{r_\alpha}+\overline{r'_\alpha u'_{\alpha i}}\right)\right)=0$$

  - Includes volume fraction-velocity correlation term.
  - Yields additional diffusion term, if we employ the eddy diffusivity hypothesis.

- **Favre Averaged Continuity Equation**

$$\frac{\partial}{\partial t}\left(\rho_\alpha \overline{r_\alpha}\right)+\frac{\partial}{\partial x_i}\left(\rho_\alpha \tilde{U}_{\alpha i}\overline{r_\alpha}\right)=0$$

  - No extra terms.
  - A mathematical simplification, not a physical one.

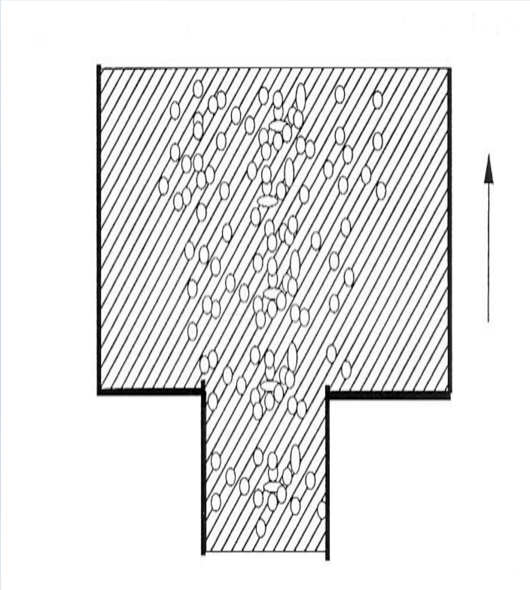
# Turbulent Dispersion Force

- Assume caused by interaction between turbulent eddies and inter-phase forces.
- Model using time average of fluctuating part of interphase momentum force.

- Restrict attention to drag force, assumed proportional to slip velocity and interfacial area density $A_{\alpha\beta}$.

$$\vec{M}_\alpha = C_{\alpha\beta}\left(\vec{U}_\beta - \vec{U}_\alpha\right) = D_{\alpha\beta} A_{\alpha\beta}\left(\vec{U}_\beta - \vec{U}_\alpha\right)$$

- Assume $D_{\alpha\beta}$ approximately constant as far as averaging procedure is concerned.



# Favre Averaged Drag Force

- Express the time averaged drag in terms of Favre averaged velocities.

$$\overline{\vec{M}_\alpha} = C_{\alpha\beta}\left(\tilde{U}_\beta - \tilde{U}_\alpha\right) + \vec{M}_\alpha^{TD}$$

- **Turbulent Dispersion Force (General Form):**

$$\vec{M}_\alpha^{TD} = -\vec{M}_\beta^{TD} = -\overline{C_{\alpha\beta}}\left(\frac{\overline{r'_\beta \vec{u}'_\beta}}{\overline{r_\beta}} - \frac{\overline{r'_\alpha \vec{u}'_\alpha}}{\overline{r_\alpha}} - \frac{\overline{a'_{\alpha\beta}\left(\vec{u}'_\beta - \vec{u}'_\alpha\right)}}{\overline{A_{\alpha\beta}}}\right)$$

- Applicable in this form to flows of arbitrary morphology, using arbitrary turbulence models.
- **Modeled Form using EDH:**

$$\vec{M}_\alpha^{TD} = -\vec{M}_\beta^{TD} = \overline{C_{\alpha\beta}}\left(\frac{\nu_{t\beta}}{\sigma_{r\beta}}\frac{\nabla\overline{r_\beta}}{\overline{r_\beta}} - \frac{\nu_{t\alpha}}{\sigma_{r\alpha}}\frac{\nabla\overline{r_\alpha}}{\overline{r_\alpha}} - \left(\frac{\nu_{t\beta}}{\sigma_{A\beta}} - \frac{\nu_{t\alpha}}{\sigma_{A\alpha}}\right)\frac{\nabla\overline{A_{\alpha\beta}}}{\overline{A_{\alpha\beta}}}\right)$$

# Polydispersed Multi-Phase Flow

- Algebraic form of area density is known:

$$A_{\alpha\beta} = \frac{6r_\beta}{d_\beta}$$

- Hence, area density-velocity correlations may be expressed in terms of volume fraction-velocity correlations:
- **General Form:**

$$\vec{M}_\alpha^{TD} = -\vec{M}_\beta^{TD} = \overline{C_{\alpha\beta}}\left(\frac{\overline{r'_\alpha \vec{u}'_\alpha}}{\overline{r_\alpha}} - \frac{\overline{r'_\beta \vec{u}_\alpha}}{\overline{r_\beta}}\right)$$

- **Eddy Diffusivity Hypothesis (EDH):**

$$\vec{M}_\alpha^{TD} = -\vec{M}_\beta^{TD} = \overline{C_{\alpha\beta}}\,\frac{\nu_{t\alpha}}{\sigma_{r\alpha}}\left(\frac{\nabla \overline{r_\beta}}{\overline{r_\beta}} - \frac{\nabla \overline{r_\alpha}}{\overline{r_\alpha}}\right)$$

# Dispersed Two-Phase Flow

- Further simplifications occur for two phases only:

$$r_\alpha + r_\beta = 1 \qquad \nabla \overline{r_\alpha} + \nabla \overline{r_\beta} = 0$$

- Modeled EDH form of the turbulent dispersion force reduces to a simple volume fraction gradient:

$$\vec{M}_\alpha^{TD} = -\vec{M}_\beta^{TD} = -\overline{C_{\alpha\beta}} \frac{\nu_{t\alpha}}{\sigma_{r\alpha}} \left( \frac{1}{\overline{\overline{r_\alpha}}} + \frac{1}{\overline{\overline{r_\beta}}} \right) \nabla \overline{r_\alpha}$$

# Comparison With Other Models

- **Imperial College Model**
  - Gosman, Lekakou, Politis, Issa, and Looney, AIChEJ 1992, Multidimensional Modeling of Turbulent Two-Phase Flows in Stirred Vessels
  - Behzadi, Issa, and Rusche (ICMF 2001), Effects of turbulence on inter-phase forces in dispersed flow.
- **Chalmers University Model**
  - Ljus (Ph. D. Thesis, 2000), On particle transport and turbulence modification in air-particle flows.
  - Johansson, Magnesson, Rundqvist and Almstedt (ICMF 2001), Study of two gas-particle flows using Eulerian/Eulerian and two-fluid models.
- **RPI Models**
  - Lopez de Bertodano, (Ph. D. Thesis,1992), Turbulent bubbly two-phase flow in a triangular duct,, RPI, New York, USA
  - Moraga, Larreteguy, Drew, and Lahey (ICMF 2001), Assessment of turbulent dispersion models for bubbly flows.

# Imperial College Model

- Idea of modeling turbulence dispersion force by Favre averaging drag term was first proposed by Gosman et al (1992).
- Behzadi et al (ICMF 2001) also consider lift and virtual mass forces, but found them insignificant.
- Equivalent to our model in the dilute limit. $r_\beta \to 0$
- Hence, validation reported by Gosman et al valid for FAD model

# Chalmers University Model

$$\vec{M}_{\alpha}^{TD} = -\vec{M}_{\beta}^{TD} = \beta_1 \nabla \overline{r_\beta} + \beta_2 \nabla k_\alpha \qquad \beta_1 = \frac{\overline{C_{\alpha\beta}}}{\overline{r_\beta}} \frac{\nu_{t\beta}}{\sigma_{d1}} \qquad \beta_2 = \frac{\overline{r_\beta} \rho_\beta}{\sigma_{d1}}$$

- Similar philosophy and derivation to our model.
- However, requires unconventionally low volume fraction Prandlt numbers, of order 0.001, to achieve reasonable agreement with experiment.
- Due to minor errors in analysis, confusing time-averaging with Favre Averaging.
- Equivalent to our model, if we identify:
- $\sigma_{d1} = \frac{\sigma_{r\alpha}}{\sigma_{\nu\beta}}$ where $\nu_{t\beta} = \frac{\nu_{t\alpha}}{\sigma_{\nu\beta}}$
- Explains low values of $\sigma_{d1}$ required to match gas-solid flow.

# RPI Models: Lopez de Bertodano

$$\vec{M}_{\alpha}^{TD} = -\vec{M}_{\beta}^{TD} = -C_{TD}\rho_{\alpha}k_{\alpha}\nabla\overline{r_{\alpha}}$$

- $C_{TD}$ is a non-dimensional empirical constant.
  - $C_{TD}$ = 0.1 to 0.5 gave reasonable results for medium sized bubbles in ellipsoidal particle regime (Lopez de Bertodano et al 1994a, 1994b).
- However, flow regimes involving small bubbles or small solid particles were found to require very different values of $C_{TD}$, up to 500.
- Revised by Lopez de Bertodano (1999).
  - Proposed that $C_{TD}$ be expressed as a function of turbulent Stokes number as follows:

$$C_{TD} = C_{\mu}^{1/4}\frac{1}{St(1+St)}$$

# RPI Models: Lopez de Bertodano

- Compare with Favre Averaged Drag (FAD) model for dispersed 2-phase flow employing EVH:

$$\vec{M}_{\alpha}^{TD} = -\vec{M}_{\beta}^{TD} = -\overline{C_{\alpha\beta}} \frac{\nu_{t\alpha}}{\sigma_{r\alpha}} \left( \frac{1}{\overline{\overline{r_{\alpha}}}} + \frac{1}{\overline{\overline{r_{\beta}}}} \right) \nabla \overline{r_{\alpha}}$$

- Substitute Eddy Viscosity Formula $\nu_{t\alpha} = C_{\mu} k_{\alpha}^{2} / \varepsilon_{\alpha}$
- Equivalent to a Lopez de Bertodano model with variable empirical constant:

$$C_{TD} = \frac{C_{\mu}}{\sigma_{r\alpha}} \frac{\overline{C_{\alpha\beta}}}{\rho_{\alpha}} \frac{k_{\alpha}}{\varepsilon_{\alpha}} \left( \frac{1}{\overline{\overline{r_{\alpha}}}} + \frac{1}{\overline{\overline{r_{\beta}}}} \right) = \frac{C_{\mu}}{0.41 \sigma_{r\alpha}} \frac{1}{St} \left( \frac{\bar{r}_{\beta}}{\bar{r}_{\alpha}} + 1 \right)$$

- Strong function of Stokes number, as expected.

# RPI Models: Carrica et al

- Requires dispersed phase volume fractions to obey a turbulent diffusion equation in limit where drag + turbulent dispersion balances body forces.

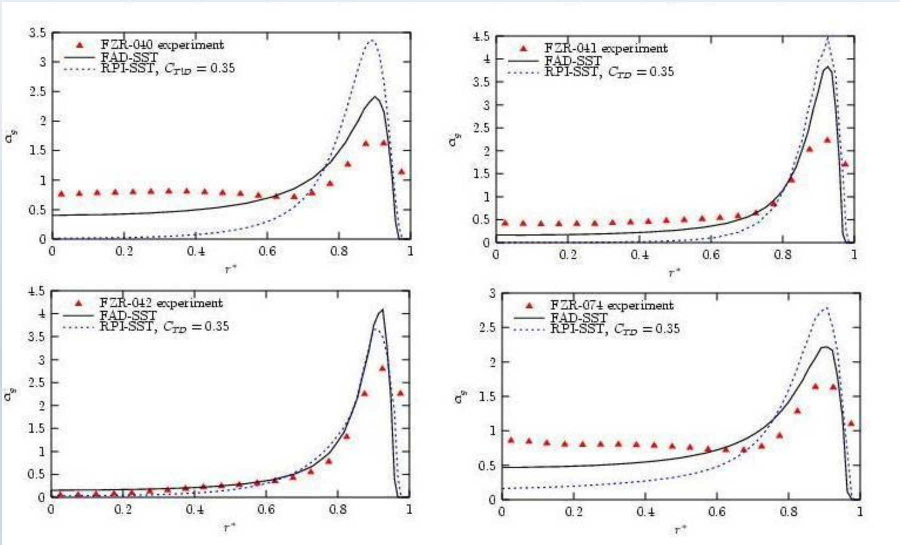
- $$\vec{M}_{\beta}^{TD} = -\frac{3}{4} C_D \frac{\rho_\alpha}{d_\beta} \left| \vec{U}_\beta - \vec{U}_\alpha \right| \frac{\nu_{t\alpha}}{\sigma_{r\alpha}} \nabla \overline{r_\beta} \qquad \beta = 2,\ldots,ND$$

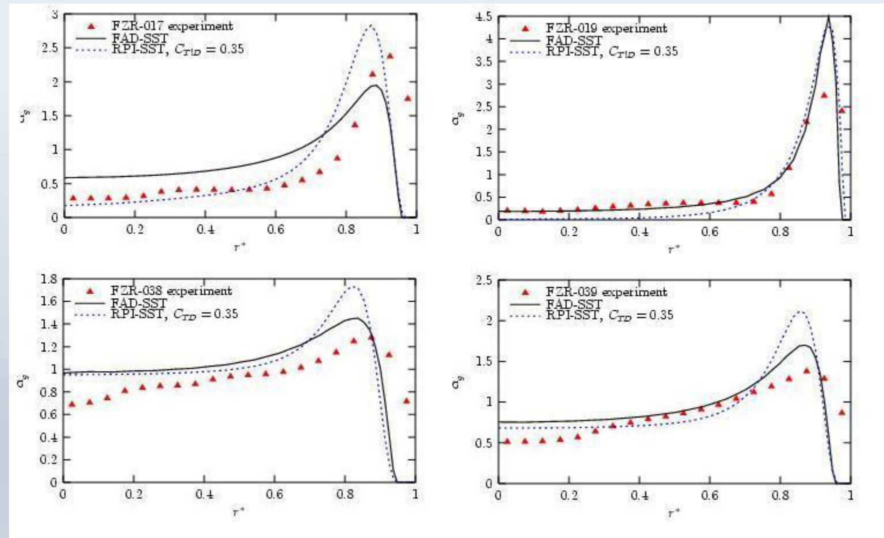
$$\vec{M}_{\alpha}^{TD} = -\sum_{\beta=2}^{ND} \vec{M}_{\beta}^{TD}$$

- Equivalent to FAD+EDH model in the following limits:
  - Two Phases Only
  - Dilute Dispersed Phase.

- Satisfactory agreement found with DNS data for dilute bubbly flows, and for bubbly mixing layer (Moraga et al, ICMF 2001).

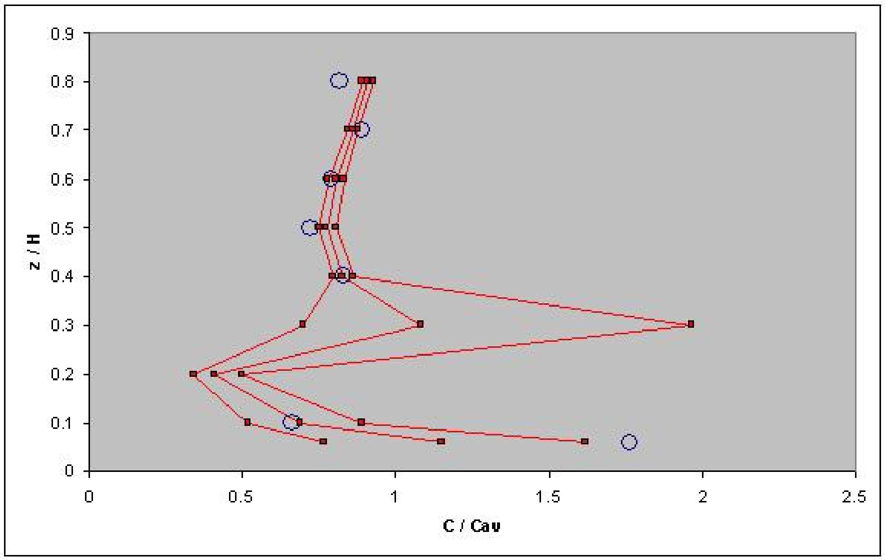
# Validation: Bubbly Flow in Vertical Pipe



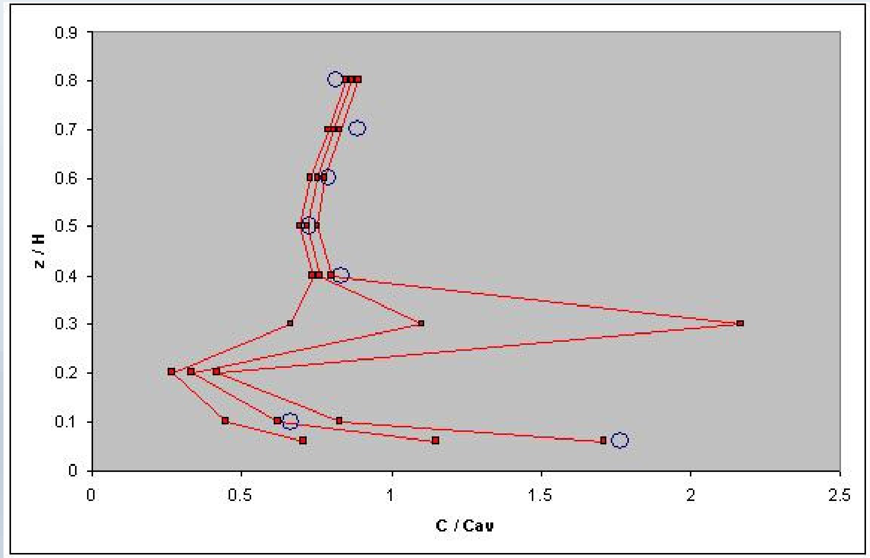




- Uses Grace Drag Law
- SST turbulence model + Sato eddy viscosity.
- Compares FAD with RPI = constant coefficient Lopez de Bertpdano model.

# Liquid-Solid Flow in Mixing Vessel



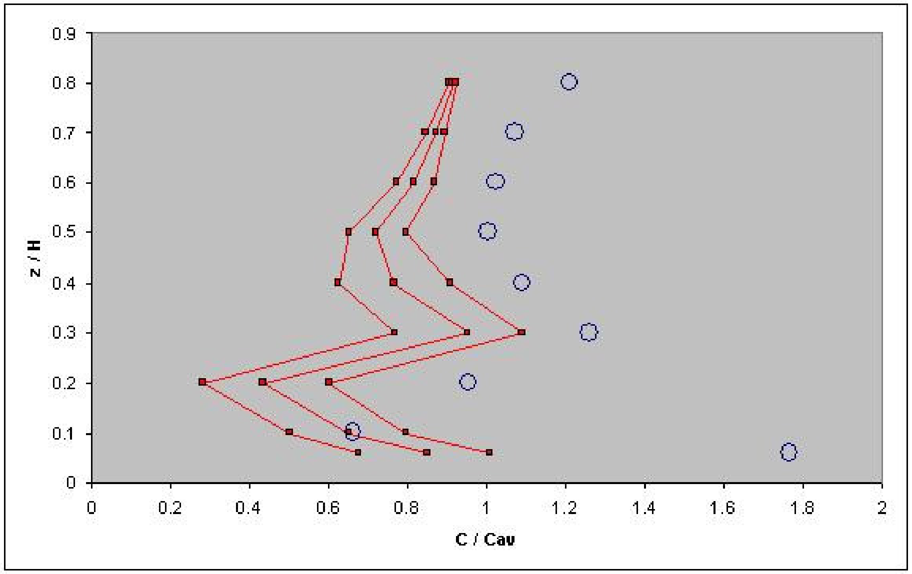

(a) r = 0.25 T; 600 microns



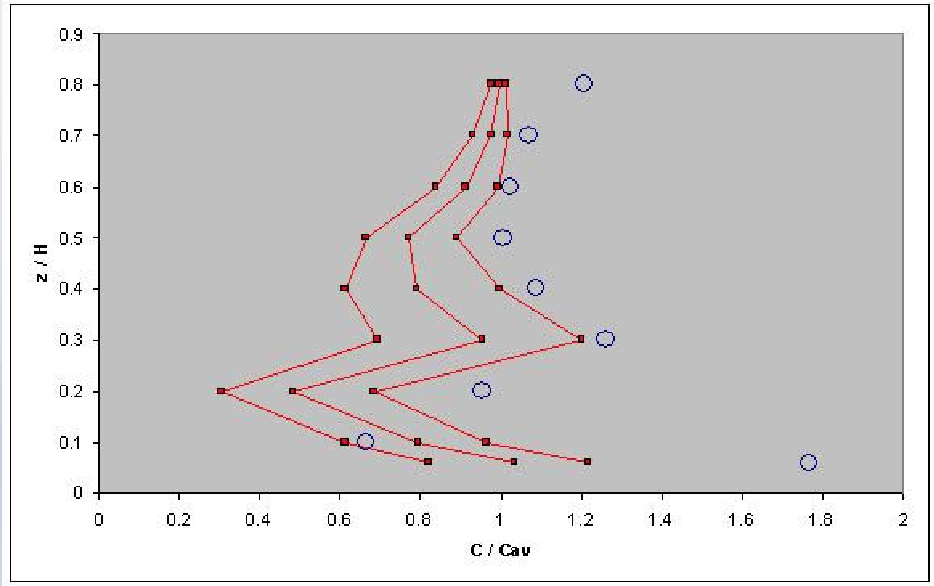

(b) r = 0.25 T; 710 microns

- Wen Yu drag correlation for dense solids.
- SST turbulence + Sato eddy viscosity.
- Three solid lines are minimum, average and maximum values of CFD results, within region ±5mm from the data point.
- Dimension representative of size of conductivity probe.

# Liquid-Solid Flow in Mixing Vessel




(c) r = 0.45T; 600 microns; xy-plane

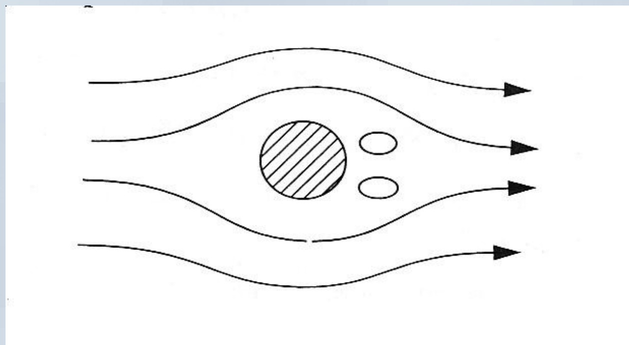



(d) r = 0.45T; 600 microns; xz-plane

- Particle volume fractions underpredicted, though correct trends are predicted.
- Similar results for 710 micron particles.

# Turbulence Modulation

- **Turbulence Enhancement**.
- Due to turbulence production in wakes behind particles.



- Averaged out by first averaging procedure (phase averaging).
- Hence, must include in first averaged equations.

# Turbulence Enhancement: Simple Models

- **Sato**: Treat particle-induced and shear-induced turbulence separately.
- Algebraic Eddy Viscosity model for phase-induced turbulence:

$$\mu_{t\alpha} = \mu_{t\alpha,si} + \mu_{t\alpha,pi}$$

$$\mu_{t\alpha,si} = C_{\mu,si}\rho_\alpha \frac{k_\alpha^2}{\varepsilon_\alpha}$$

$$\mu_{t,pi} = C_\mu \rho_c r_d d_p \left| U_c - U_d \right|$$

- **Modifed k-ε Models**
- Lump particle-induced and shear-induced turbulence together.

$$\mu_{t\alpha} = C_\mu \rho_\alpha \frac{k_\alpha^2}{\varepsilon_\alpha}$$

- Add additional production terms to shear-induced k-e equations, e.g. Lee at al

$$P_{k\alpha} = C_{\alpha\beta}\left(\vec{U}_\beta - \vec{U}_\alpha\right)^2$$

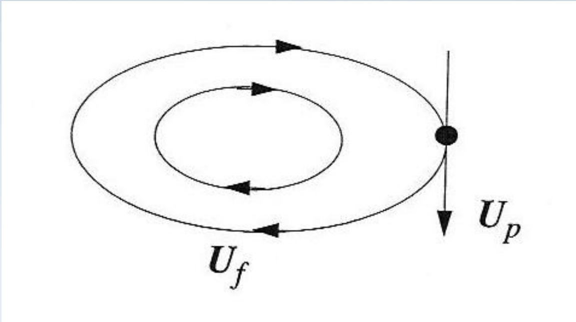
$$P_{\varepsilon\alpha} = \frac{\varepsilon_\alpha}{k_\alpha} C_1 P_{k\alpha}$$

# Turbulence Reduction

- Energy transferred from turbulent eddies to particles by acceleration of particles due to drag.
- **Turbulence-drag interaction**, like dispersion.
- Interaction with shear-induced turbulence, so appears as additional source terms in **2nd averaged k-equation**

$$S_{k\alpha} = \overline{\vec{M}_\alpha \cdot \vec{u}'_\alpha}$$

$$S_{k\beta} = \overline{\vec{M}_\beta \cdot \vec{u}'_\beta}$$



# Turbulence Reduction: Simple Model

- **Chen-Wood**: Consider drag only, and treat $C_{\alpha\beta}$ as constant in averaging procedure:

$$S_{k\alpha} = \overline{C_{\alpha\beta}\left(\vec{U}_\beta - \vec{U}_\beta\right)\cdot \vec{u}'_\alpha} = \overline{C_{\alpha\beta}}\left(\overline{\vec{u}'_\alpha \vec{u}'_\beta} - \overline{\vec{u}'_\alpha \vec{u}'_\alpha}\right) = \overline{C_{\alpha\beta}}\left(k_{\alpha\beta} - 2k_\alpha\right)$$

$$S_{k\beta} = \overline{C_{\alpha\beta}\left(\vec{U}_\alpha - \vec{U}_\beta\right)\cdot \vec{u}'_\beta} = \overline{C_{\alpha\beta}}\left(\overline{\vec{u}'_\alpha \vec{u}'_\beta} - \overline{\vec{u}'_\beta \vec{u}'_\beta}\right) = \overline{C_{\alpha\beta}}\left(k_{\alpha\beta} - 2k_\beta\right)$$

- $k_{\alpha\beta} = \overline{\vec{u}'_\alpha \vec{u}'_\beta}$ = **velocity covariance**

- Sum of sources is **negative** $S_{k\alpha} + S_{k\beta} = -\overline{C_{\alpha\beta}}\,\overline{\left(\vec{u}'_\alpha - \vec{u}'_\beta\right)^2} \le 0$
- Hence, can only model turbulence **reduction.**

- Requires model for velocity covariance:
- **Chen-Wood**: $u'_d = C u'_c$ $\overline{u'_{ic}u'_{jd}} = C\,\overline{u'_{ic}u'_{jc}}$

# Turbulence Enhancement: Proposed Double Averaged Approach

- Treat phase-induced and shear-induced separately, as in Sato model.
- Solve separate transport equations for phase-induced and shear-induced turbulence.
- *k-l* model for phase-induced turbulence: (Lopez de Bertodano et al)

$$\mu_{t\alpha,pi} = C_\mu \rho_\alpha \sqrt{k_{\alpha,pi}}\, l_{t\alpha,pi}$$

$$\frac{\partial}{\partial t}\left(\tilde{\rho}_\alpha k_{\alpha,pi}\right) + \frac{\partial}{\partial x_i}\left(\tilde{\rho}_\alpha U_\alpha k_{\alpha,pi} - \frac{\mu_{t\alpha,pi}}{\sigma_{,pik}}\frac{\partial k_{\alpha,pi}}{\partial x_i}\right) = \frac{\tilde{\rho}_\alpha}{\tau_{k,pi}}\left(k_{pi,\infty} - k_{\alpha,pi}\right)$$

- Choose

$$k_{pi,\infty} = \frac{1}{4} r_\alpha \left|\vec{U}_\beta - \vec{U}_\alpha\right|^2 \qquad l_{t\alpha,pi} \propto d_\beta$$

- Matches Sato: $\mu_{t,pi} = C_\mu \rho_c r_d d_p \left|U_c - U_d\right|$
- Choose time-scale $\tau_{k,pi}$ to match Kataoka-Serizawa production:

$$\frac{\tilde{\rho}_\alpha k_{pi,\infty}}{\tau_{k,pi}} = C_{\alpha\beta}\left(\vec{U}_\beta - \vec{U}_\alpha\right)^2$$

- Hence, $\tau_{k,pi}$ proportional to particle relaxation time.
- Time Averaged $k_{pi}$ equation introduces additional terms involving turbulence dispersion force.
- Hence, affected by volume fraction gradients.

# Turbulence Reduction: Proposed Double Average Approach

- As for Chen-Wood, but take area density fluctuations into account in averaging procedure.
- Hence, additional source terms in shear-induced k-equation:
- $S_{k\alpha} = D_{\alpha\beta}\overline{A_{\alpha\beta}\left(\vec{U}_{\beta} - \vec{U}_{\beta}\right)\cdot \vec{u}'_{\alpha}} = \overline{C_{\alpha\beta}}\left(k_{\alpha\beta} - 2k_{\alpha}\right)$ + additional terms
- Additional terms proportional to $\overline{C_{\alpha\beta}}\left(\tilde{U}_{\beta} - \tilde{U}_{\alpha}\right)\cdot \dfrac{\mu_t}{\sigma_r}\nabla r_{\alpha}$
- Hence, also affected by volume fraction gradients.
- Consider better models for velocity covariance, e.g. transport equation.

# Conclusions

- Double Averaging Approach yields a natural model for turbulence dispersion, with wide degree of universality.
- Implemented as default model for turbulence dispersion in CFX-5.7 (2004).
- Also produces potentially fruitful approaches to turbulence modulation.
- Topics for further investigation:
  - How is the model affected by taking into account non-linear dependence of drag on slip velocity?
  - How is the model affected by taking into account volume fraction dependence of the drag coefficient?
  - second order closure models.
  - separated flows.