Turbulence Dispersion Force
– Physics, Model Derivation and Evaluation–

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Turbulent dispersion (TD)

A result of the (dispersed phase) particle–eddy (continuous phase) interaction

Important for dispersed phase with small $St$

Turbulent mass diffusion $\left( \alpha_k' u_k' \right)$ and interphase momentum transfer

Turbulent mass diffusion is usually modeled by a mass diffusion term

The interphase momentum transfer due to turbulent dispersion is often separated as an “interfacial force”, to be calculated from the temporal correlation of the interfacial force fluctuations. Usually, only the drag contribution is essential.
Eulerian model – mass diffusion

• Reynolds averaging

\[
\frac{\partial}{\partial t}(\rho_k \alpha_k) + \nabla \cdot (\rho_k \alpha_k \mathbf{U}_k) = 0 \quad \text{applying } \phi = \bar{\phi} + \phi' \quad \Rightarrow \quad (1)
\]

\[
\frac{\partial}{\partial t}(\rho_k \bar{\alpha}_k) + \nabla \cdot (\rho_k \bar{\alpha}_k \bar{\mathbf{U}}_k) = -\nabla \cdot (\rho_k \alpha_k \mathbf{u}_k') \quad \text{with } \frac{\alpha_k}{\bar{\alpha}_k} \mathbf{u}_k' \sim -\frac{\nu_{k,t}}{\sigma_k} \nabla \bar{\alpha}_k \quad (2)
\]

• Adopting Favré-averaged velocity

\[
\tilde{\mathbf{U}}_k = \frac{\alpha_k \mathbf{U}_k}{\bar{\alpha}_k} = \bar{\mathbf{U}}_k + \frac{\alpha_k}{\bar{\alpha}_k} \mathbf{u}_k' \quad (3)
\]

\[
\mathbf{U}_k = \tilde{\mathbf{U}}_k + \mathbf{u}_k'' + \mathbf{u}_k', \quad \mathbf{u}_k'' = -\frac{\alpha_k}{\bar{\alpha}_k} \mathbf{u}_k' \neq 0 \quad (4)
\]

\[
\frac{\partial}{\partial t}(\rho_k \bar{\alpha}_k) + \nabla \cdot (\rho_k \bar{\alpha}_k \tilde{\mathbf{U}}_k) = 0 \quad (5)
\]

• Using Favré-averaged velocity simplifies the equation system
Eulerian model – turbulent dispersion force

The Favré-Averaged Drag Force (FAD) model,

\[
\text{Drag: } \mathbf{F}_D = C_{fp}(\mathbf{U}_p - \mathbf{U}_f) = \frac{1}{8} C_D \rho_f |\mathbf{U}_p - \mathbf{U}_f| D_{fp} \frac{6 \alpha_f}{d_p} (\mathbf{U}_p - \mathbf{U}_f) \quad (6)
\]

Reynolds averaging: \[
\overline{\mathbf{F}_D} = \overline{C_{fp}} (\overline{\mathbf{U}}_p - \overline{\mathbf{U}}_f) + \mathbf{F}_{TD} \quad (7)
\]

\[
\mathbf{F}_{TD} = \overline{C_{fp}} \left( \frac{\alpha_f' \mathbf{u}_f'}{\alpha_f} - \frac{\alpha_p' \mathbf{u}_f'}{\alpha_p} \right) \quad \text{EDH} \quad \overline{C_{fp}} \frac{\nu_{f,t}}{\sigma_f} \left( \frac{\nabla \alpha_p}{\alpha_p} - \frac{\nabla \alpha_f}{\alpha_f} \right) \quad (8)
\]

For details refer to

Lagrangian method

tracking trajectories of each particle, parcel

\[ x_{p}^{n+1} = x_{p}^{n} + V_{p}^{n} \delta t \]  \hspace{1cm} (9)

\[ \frac{V_{p}^{n+1} - V_{p}^{n}}{\delta t} = \left( \frac{3 \rho_{f} C_{D}}{4 \rho_{p} d_{p}} |V_{p} - V_{f}| \right)^{n} (V_{p} - V_{f}) + \frac{1}{\rho_{p} \delta V} F_{\text{other}} \]  \hspace{1cm} (10)

construct fluid fluctuating velocity for particle-eddy interaction

\[ V_{f}^{n+1} = U_{f}(x_{p}^{n+1}, t) + v_{f}^{n+1}, \quad v_{f}^{n+1} = av_{f}^{n} + be^{n}, \text{ where} \]  \hspace{1cm} (11)

\[ a = \exp \left( -\frac{\delta t}{T_{L}^{*}} \right), \quad b = \left( \frac{2k_{f}}{3} \right)^{\frac{1}{2}} \sqrt{1 - a^{2}}, \quad |e|^{n} \in N(0, 1) \]  \hspace{1cm} (12)
A new TD force model derivation (1)

Ensemble average for a cell $\Delta V$ at time $t_j$

\[
F_D(t_j) = \frac{1}{\Delta V} \sum_{i=1}^{n(t_j)} f^i_D(t_j) = \frac{1}{\Delta V} \sum_{i=1}^{n(t_j)} 3 \frac{C_i^D}{\frac{d^i_p}{V_p^i - V_f^i}} (V_p^i - V_f^i) \delta V^i
\]  

(13)

Time averaged drag

\[
\overline{F_D} = \frac{1}{\Delta V T_E} \sum_{j=-N}^{N} \sum_{i=1}^{n(t_j)} 3 \frac{C_i^D}{\frac{d^i_p}{V_p^i - V_f^i}} (V_p^i - V_f^i) \delta V^i \delta t
\]

(14)

$T_E$ (macro time scale) $\gg \tau_e = C \frac{k}{\epsilon}$ (eddy life time); $\delta t \ll \tau_e$

\[
\begin{array}{cccc}
\begin{array}{c}
\Delta V
\end{array} & \begin{array}{c}
n(t_{-N})
\end{array} & \cdots & \begin{array}{c}
n(t_{-j})
\end{array} \\
\begin{array}{c}
t_{-N} = t - T_E/2
\end{array} & \begin{array}{c}
t_{-j} = t - j \delta t
\end{array} & \cdots & \begin{array}{c}
t_0 = t
\end{array} \\
\end{array}
\begin{array}{cc}
\begin{array}{c}
n(t_0)
\end{array} & \begin{array}{c}
n(t_j)
\end{array} \\
t_0 = t & t_j = t + j \delta t
\end{array}
\begin{array}{c}
\begin{array}{c}
n(t_N)
\end{array}
\end{array}
\begin{array}{c}
t_N = t + T_E/2
\end{array}
\end{array}
\]
Model derivation (2)

Converting into the Eulerian frame

- Eulerian variables

\[
\alpha_p(t_j) = \frac{1}{\Delta V} \sum_{i=1}^{n(t_j)} \delta V^i
\]

\[
\Phi(t_j) = \frac{\sum_{i=1}^{n(t_j)} \Phi^i \delta V^i}{\sum_{i=1}^{n(t_j)} \delta V^i} = \frac{1}{\Delta V} \frac{\sum_{i=1}^{n(t_j)} \Phi^i \delta V^i}{\alpha_p(t_j)}
\]

- Reynolds averaging operator

\[
\overline{\Phi(t)} = \frac{1}{T_E} \int_{t-T_E/2}^{t+T_E/2} \Phi(\theta) d\theta = \frac{1}{T_E} \sum_{j=-N}^{N} \Phi(t_j) \delta t
\]

- Averaging the drag force

\[
\overline{F_D} = \overline{\alpha_p f_D} = \overline{\alpha_p f_D} + \overline{\alpha_p' f_D'}
\]

(mean drag + turbulent dispersion force)
Derivation details (1)

\[ F_D = \frac{3}{4} \rho_f \frac{C_D}{d_p} |U_p - U_f| (U_p - U_f) \alpha_p \]

\[ \approx \frac{3}{4} \rho_f \frac{C_D}{d_p} |(U_p - U_f)| \alpha_p (U_p - U_f) \]

\[ = D |U_p - U_f| \left( \alpha_p \tilde{U}_p - \alpha_p \tilde{U}_f \right) \]

\[ = D |U_p - U_f| \left( \alpha_p \tilde{U}_p - \alpha_p \bar{U}_f - \alpha'_p u'_f \right) \]

\[ = D \alpha_p |U_p - U_f| (\tilde{U}_p - \tilde{U}_f) + D |U_p - U_f| \left( \frac{\alpha_p}{\alpha_f} \frac{\alpha'_f u'_f}{u'_f} - \alpha'_p u'_f \right) \]

(19) turbulent dispersion force \( F_{TD} \)
Derivation details (2)

Eddy viscosity hypothesis (EVH)

\[
\alpha' u_f' = -\frac{\nu_{f,t}}{\sigma_f} \nabla \alpha \quad \Rightarrow 
\bar{u}_f'' = \frac{\nu_{f,t}}{\sigma_f \bar{\alpha}} \nabla \bar{\alpha} 
\]  

(20)

Model expression (for the continuous phase)

\[
F_{TD} \approx \frac{3}{4} \rho_f \frac{C_D}{d_p} \frac{\nu_{f,t}}{\sigma_f} |U_p - U_f| \left( \nabla \bar{\alpha}_p - \frac{\bar{\alpha}_p}{\bar{\alpha}_f} \nabla \bar{\alpha}_f \right) \]  

(21)

Results for two-fluid model, \( \bar{\alpha}_p + \bar{\alpha}_f = 1 \)

\[
F_{TD} \approx \frac{3}{4} \rho_f \frac{C_D}{d_p} \frac{\nu_{f,t}}{\sigma_f} |U_p - U_f| \frac{\nabla \bar{\alpha}_p}{\bar{\alpha}_f} 
\]  

(22)
Remarks

The present derivation illustrates the physics of the turbulent dispersion

The present derivation explains why double average makes sense

Lagrangian evaluation of turbulent dispersion is very expensive. This derivation might provide a theoretical foundation for a deterministic TD force model for the Lagrangian solver
Evaluation
FZR MTLoop test facility

Air-water system, isothermal

Inner pipe diameter $D = 51.2$ mm

Wire mesh sensor measurements

Test section from gas injection: $L = 0.03$ to $3.03$ m

Injection nozzle arrangement:
Test case definition
Two-fluid model Evaluation

For details refer to

Th. Frank, J.-M. Shi, and A. Burns, 3rd International Symposium on Two-Phase Flow Modeling and Experimentation, Pisa, Italy, 22-24 September, 2004
Poly-dispersed model Evaluation

- **Stationary, axisym., bubbly flow** at the upper test section ($L/D = 59.2$)

- Data from measurements: superficial velocities, mean bubble diameter, local gas volume fraction

<table>
<thead>
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<th>Index</th>
<th>Ul [m/s]</th>
<th>Ug [m/s]</th>
<th>Air VF [%]</th>
<th>Air 1 dp [mm]</th>
<th>Air 1 VF [%]</th>
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Ul, Ug–superficial velocity, dp–diameter [mm], VF–gas volume fraction.
Results from poly-dispersed model

\begin{align*}
\alpha^* &= 10^{-0.4r^* + 0.8} \\
\alpha^* &= 10^{-0.6r^* + 0.6} \\
\alpha^* &= 10^{-0.8r^*} \\
\end{align*}
**TD coefficient, FZR070,**

\[
C_{TD} = \frac{3}{4} \rho_f \frac{C_D}{d_p} \frac{\nu_{f,t}}{\sigma_f} |U_p - U_f|
\]
**TD coefficient, FZR110,** \( C_{TD} = \frac{3}{4} \rho f \frac{C_D}{d_p} \frac{\nu f_t}{\sigma_f} |U_p - U_f| \)