Turbulence Dispersion Force
– Physics, Model Derivation and Evaluation–

J.-M. Shi[1], T. Frank[2], A. Burns[3]

1 Institute of Safety Research, FZ Rossendorf

shi@fz-rossendorf.de

2 ANSYS CFX Germany 3 ANSYS CFX

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Institut für Sicherheitsforschung

Contents

Turbulent dispersion (TD)

A result of the (dispersed phase) particle–eddy (continuous phase) interaction

Important for dispersed phase with small St

Turbulent mass diffusion ($\overline{\alpha_k^{'} \mathbf{u}_\mathbf{k}^{'}}$) and interphase momentum transfer

Turbulent mass diffusion is usually modeled by a mass diffusion term

The interphase momentum transfer due to turbulent dispersion is often separated as an "interfacial force", to be calculated from the temperal correlation of the interfacial force fluctuations. Usually, only the drag contribution is essential.

Eulerian model – mass diffusion

- Reynolds averaging

$$\frac{\partial}{\partial t}(\rho_k \alpha_k) + \nabla \cdot (\rho_k \alpha_k \mathbf{U_k}) = 0 \quad \text{applying } \phi = \overline{\phi} + \phi' \quad \Rightarrow \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_k \overline{\alpha}_k) + \nabla \cdot (\rho_k \overline{\alpha}_k \overline{\mathbf{U}}_\mathbf{k}) = -\nabla \cdot (\rho_k \overline{\alpha_k' \mathbf{u_k'}}) \quad \text{with } \overline{\alpha_k' \mathbf{u_k'}} \sim -\frac{\nu_{k,t}}{\sigma_k} \nabla \overline{\alpha}_k \tag{2}$$

- Adopting Favré-averaged velocity

$$\tilde{\mathbf{U}}_\mathbf{k} = \frac{\overline{\alpha_k \mathbf{U_k}}}{\overline{\alpha}_k} = \overline{\mathbf{U}}_\mathbf{k} + \frac{\overline{\alpha_k' \mathbf{u_k'}}}{\overline{\alpha}_k} \tag{3}$$

$$\mathbf{U_k} = \tilde{\mathbf{U}}_\mathbf{k} + \overline{\mathbf{u_k''}} + \mathbf{u_k'}, \quad \overline{\mathbf{u_k''}} = -\frac{\overline{\alpha_k' \mathbf{u_k'}}}{\overline{\alpha}_k} \neq 0 \tag{4}$$

$$\frac{\partial}{\partial t}(\rho_k \overline{\alpha}_k) + \nabla \cdot (\rho_k \overline{\alpha}_k \tilde{\mathbf{U}}_\mathbf{k}) = 0 \tag{5}$$

- Using Favré-averaged velocity simplifies the equation system

Eulerian model – turbulent dispersion force

The Favré-Averaged Drag Force (FAD) model,

Drag: $$\mathbf{F_D} = C_{fp}(\mathbf{U_p} - \mathbf{U_f}) = \underbrace{\frac{1}{8} C_D \rho_f |\mathbf{U_p} - \mathbf{U_f}|}_{D_{fp}} \underbrace{\frac{6\,\alpha_f}{d_p}}_{A_{fp}} (\mathbf{U_p} - \mathbf{U_f}) \qquad (6)$$

Reynolds averaging: $$\overline{\mathbf{F_D}} = \overline{C_{fp}} \left(\tilde{\mathbf{U}}_\mathbf{p} - \tilde{\mathbf{U}}_\mathbf{f} \right) + \mathbf{F_{TD}} \qquad (7)$$

$$\mathbf{F_{TD}} = \overline{C_{fp}} \left(\frac{\overline{\alpha'_f \mathbf{u}'_\mathbf{f}}}{\overline{\alpha_f}} - \frac{\overline{\alpha'_p \mathbf{u}'_\mathbf{f}}}{\overline{\alpha_p}} \right) \xrightarrow{EDH} \overline{C_{fp}} \frac{\nu_{f,t}}{\sigma_f} \left(\frac{\nabla \overline{\alpha_p}}{\overline{\alpha_p}} - \frac{\nabla \overline{\alpha_f}}{\overline{\alpha_f}} \right) \qquad (8)$$

For details refer to

A. Burns, T. Frank, I. Hamill and J.-M. Shi, ICMF2004, Paper No. 392

Lagrangian method

tracking trajectories of each particle, parcel

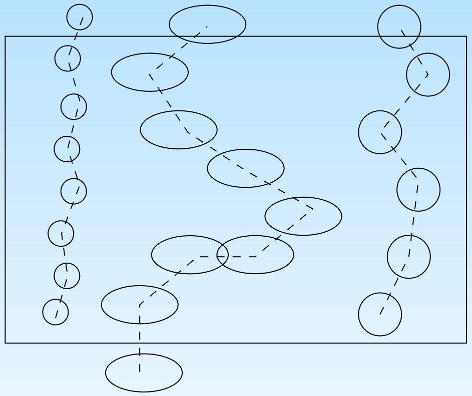
$$\mathbf{x}_\mathbf{p}^{n+1} = \mathbf{x}_\mathbf{p}^{n} + \mathbf{V}_\mathbf{p}^{n}\delta t \quad (9)$$

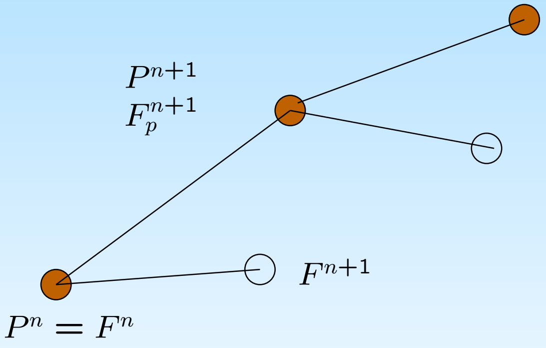
$$\frac{\mathbf{V}_\mathbf{p}^{n+1} - \mathbf{V}_\mathbf{p}^{n}}{\delta t} = \left(\frac{3}{4}\frac{\rho_f}{\rho_p}\frac{C_D}{d_p}|\mathbf{V}_\mathbf{p} - \mathbf{V}_\mathbf{f}|\right)^n (\mathbf{V}_\mathbf{p} - \mathbf{V}_\mathbf{f}) + \frac{1}{\rho_p \delta V}\mathbf{F}_\mathbf{other} \quad (10)$$

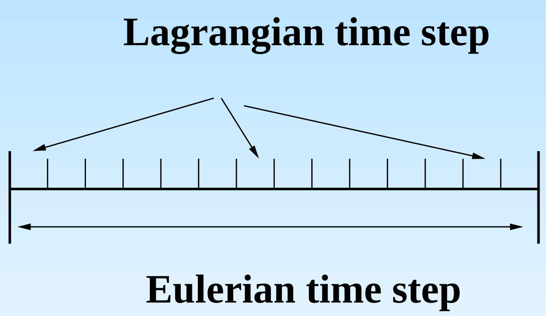
construct fluid fluctuating velocity for particle-eddy interaction

$$\mathbf{V}_\mathbf{f}^{n+1} = \mathbf{U}_\mathbf{f}(\mathbf{x}_\mathbf{p}^{n+1}, t) + \mathbf{v}_\mathbf{f}^{n+1} \quad , \quad \mathbf{v}_\mathbf{f}^{n+1} = a\mathbf{v}_\mathbf{f}^{n} + b\mathbf{e}^n \text{ , where} \quad (11)$$

$$a = \exp\left(-\frac{\delta t}{T_L^*}\right), \quad b = \left(\frac{2k_f}{3}\right)^{\frac{1}{2}}\sqrt{1-a^2}, \quad |\mathbf{e}|^n \in N(0,1) \quad (12)$$







A new TD force model derivation (1)

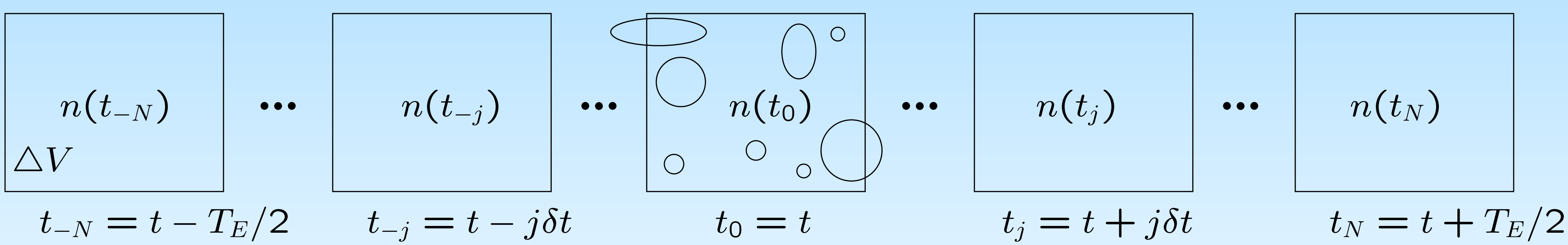
Ensemble average for a cell $\triangle V$ at time t_j

$$\mathbf{F_D}(t_j) = \frac{1}{\triangle V}\sum_{i=1}^{n(t_j)} \mathbf{f_D^i}(t_j) = \frac{1}{\triangle V}\sum_{i=1}^{n(t_j)} \frac{3}{4}\rho_f \frac{C_D^i}{d_p^i}|\mathbf{V_p^i} - \mathbf{V_f^i}|(\mathbf{V_p^i} - \mathbf{V_f^i})\delta V^i \quad (13)$$

Time averaged drag

$$\overline{\mathbf{F_D}} = \frac{1}{\triangle V}\frac{1}{T_E}\sum_{j=-N}^{N}\sum_{i=1}^{n(t_j)} \frac{3}{4}\rho_f \frac{C_D^i}{d_p^i}|\mathbf{V_p^i} - \mathbf{V_f^i}|(\mathbf{V_p^i} - \mathbf{V_f^i})\delta V^i \delta t \quad (14)$$

T_E (macro time scale) $\gg \tau_e = C\,\frac{k}{\epsilon}$ (eddy life time); $\delta t \ll \tau_e$



Model derivation (2)

Converting into the Eulerian frame

- Eulerian variables

$$\alpha_p(t_j) = \frac{1}{\triangle V} \sum_{i=1}^{n(t_j)} \delta V^i \tag{15}$$

$$\Phi(t_j) = \frac{\sum_{i=1}^{n(t_j)} \Phi^i \delta V^i}{\sum_{i=1}^{n(t_j)} \delta V^i} = \frac{1}{\triangle V} \frac{\sum_{i=1}^{n(t_j)} \Phi^i \delta V^i}{\alpha_p(t_j)} \tag{16}$$

- Reynolds averaging operator

$$\overline{\Phi(t)} = \frac{1}{T_E} \int_{t-T_E/2}^{t+T_E/2} \Phi(\theta) \mathrm{d}\theta = \frac{1}{T_E} \sum_{j=-N}^{N} \Phi(t_j) \delta t \tag{17}$$

- Averaging the drag force

$$\overline{\mathbf{F_D}} = \overline{\alpha_p \mathbf{f_D}} = \underbrace{\overline{\alpha_p}\,\overline{\mathbf{f_D}}}_{\text{mean drag}} + \underbrace{\overline{\alpha_p' \mathbf{f_D'}}}_{\text{turbulent dispersion force}} \tag{18}$$

Derivation details (1)

$$\begin{aligned}
\overline{\mathbf{F_D}} &= \overline{\frac{3}{4}\rho_f \frac{C_D}{d_p} |\mathbf{U_p} - \mathbf{U_f}| \left(\mathbf{U_p} - \mathbf{U_f}\right) \alpha_p} \\
&\approx \underbrace{\frac{3}{4}\rho_f \frac{\overline{C_D}}{\overline{d_p}}}_{D} \overline{|\left(\mathbf{U_p} - \mathbf{U_f}\right)|}\; \overline{\alpha_p \left(\mathbf{U_p} - \mathbf{U_f}\right)} \\
&= D\overline{|\mathbf{U_p} - \mathbf{U_f}|} \;\left(\overline{\alpha_p \mathbf{U_p}} - \overline{\alpha_p \mathbf{U_f}}\right) \\
&= D\overline{|\mathbf{U_p} - \mathbf{U_f}|} \;\left(\overline{\alpha_p}\tilde{\mathbf{U}}_\mathbf{p} - \overline{\alpha_p}\;\overline{\mathbf{U}}_\mathbf{f} - \overline{\alpha_p^{'} \mathbf{u}_\mathbf{f}^{'}}\right) \\
&= D\overline{|\mathbf{U_p} - \mathbf{U_f}|} \;\left[\overline{\alpha_p}\left(\tilde{\mathbf{U}}_\mathbf{p} - \tilde{\mathbf{U}}_\mathbf{f}\right) + \overline{\alpha_p}\frac{\overline{\alpha_f^{'} \mathbf{u}_\mathbf{f}^{'}}}{\overline{\alpha_f}} - \overline{\alpha_p^{'} \mathbf{u}_\mathbf{f}^{'}}\right] \\
&= D\overline{\alpha_p}\overline{|\mathbf{U_p} - \mathbf{U_f}|} \;\left(\tilde{\mathbf{U}}_\mathbf{p} - \tilde{\mathbf{U}}_\mathbf{f}\right) + \underbrace{D\overline{|\mathbf{U_p} - \mathbf{U_f}|}\left(\frac{\overline{\alpha_p}}{\overline{\alpha_f}}\overline{\alpha_f^{'} \mathbf{u}_\mathbf{f}^{'}} - \overline{\alpha_p^{'} \mathbf{u}_\mathbf{f}^{'}}\right)}_{\text{turbulent dispersion force } \mathbf{F_{TD}}} \qquad (19)
\end{aligned}$$

Derivation details (2)

Eddy viscosity hypothesis (EVH)

$$\overline{\alpha' \mathbf{u}'_\mathbf{f}} = -\frac{\nu_{f,t}}{\sigma_f}\nabla\overline{\alpha} \qquad \Rightarrow \qquad \overline{\mathbf{u}''_\mathbf{f}} = \frac{\nu_{f,t}}{\sigma_f\,\overline{\alpha}}\nabla\overline{\alpha} \tag{20}$$

Model expression (for the continuous phase)

$$\mathbf{F_{TD}} \approx \underbrace{\frac{3}{4}\rho_f\frac{\overline{C_D}}{\overline{d_p}}\frac{\nu_{f,t}}{\sigma_f}\overline{|\mathbf{U_p}-\mathbf{U_f}|}}_{C_{TD}}\left(\nabla\overline{\alpha_p} - \frac{\overline{\alpha_p}}{\overline{\alpha_f}}\nabla\overline{\alpha_f}\right) \tag{21}$$

Results for two-fluid model, $\alpha_p + \alpha_f = 1$

$$\mathbf{F_{TD}} \approx \frac{3}{4}\rho_f\frac{\overline{C_D}}{\overline{d_p}}\frac{\nu_{f,t}}{\sigma_f}\overline{|\mathbf{U_p}-\mathbf{U_f}|}\frac{\nabla\overline{\alpha_p}}{\overline{\alpha_f}} \tag{22}$$

Remarks

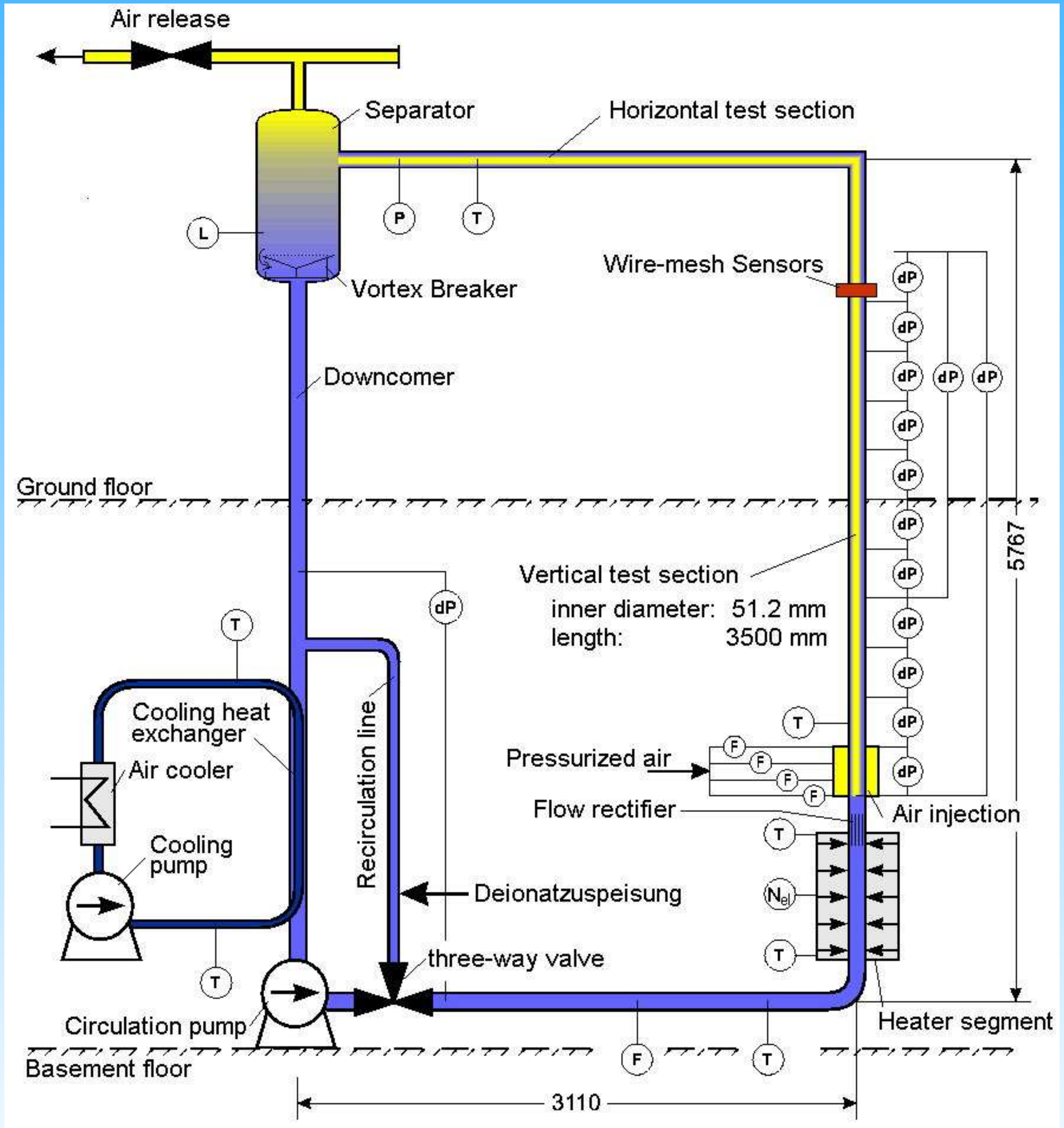
The present derivation illustrates the physics of the turbulent dispersion

The present derivation explains why double average makes sense

Lagrangian evaluation of turbulent dispersion is very expensive. This derivation might provide a theoretical foundation for a deterministic TD force model for the Lagrangian solver

Evaluation

FZR MTLoop test facility



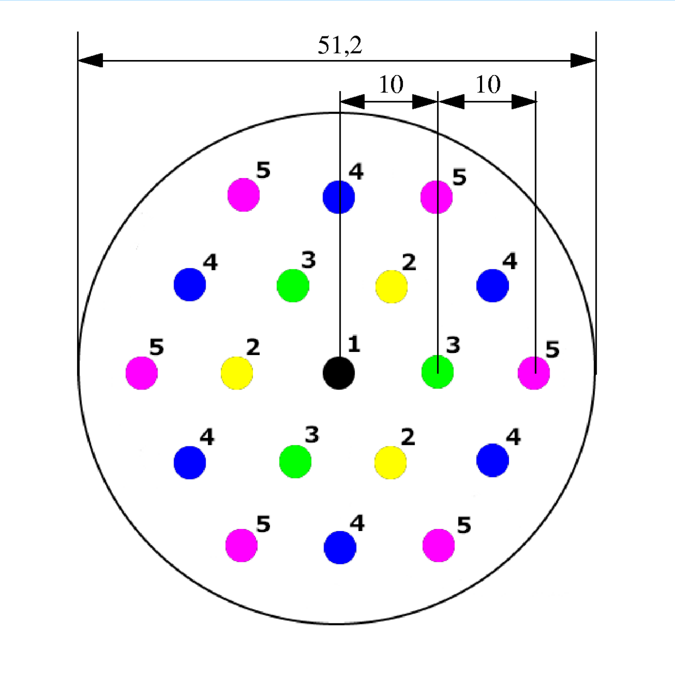
Air-water system, isothermal

Inner pipe diameter $D = 51.2$ mm

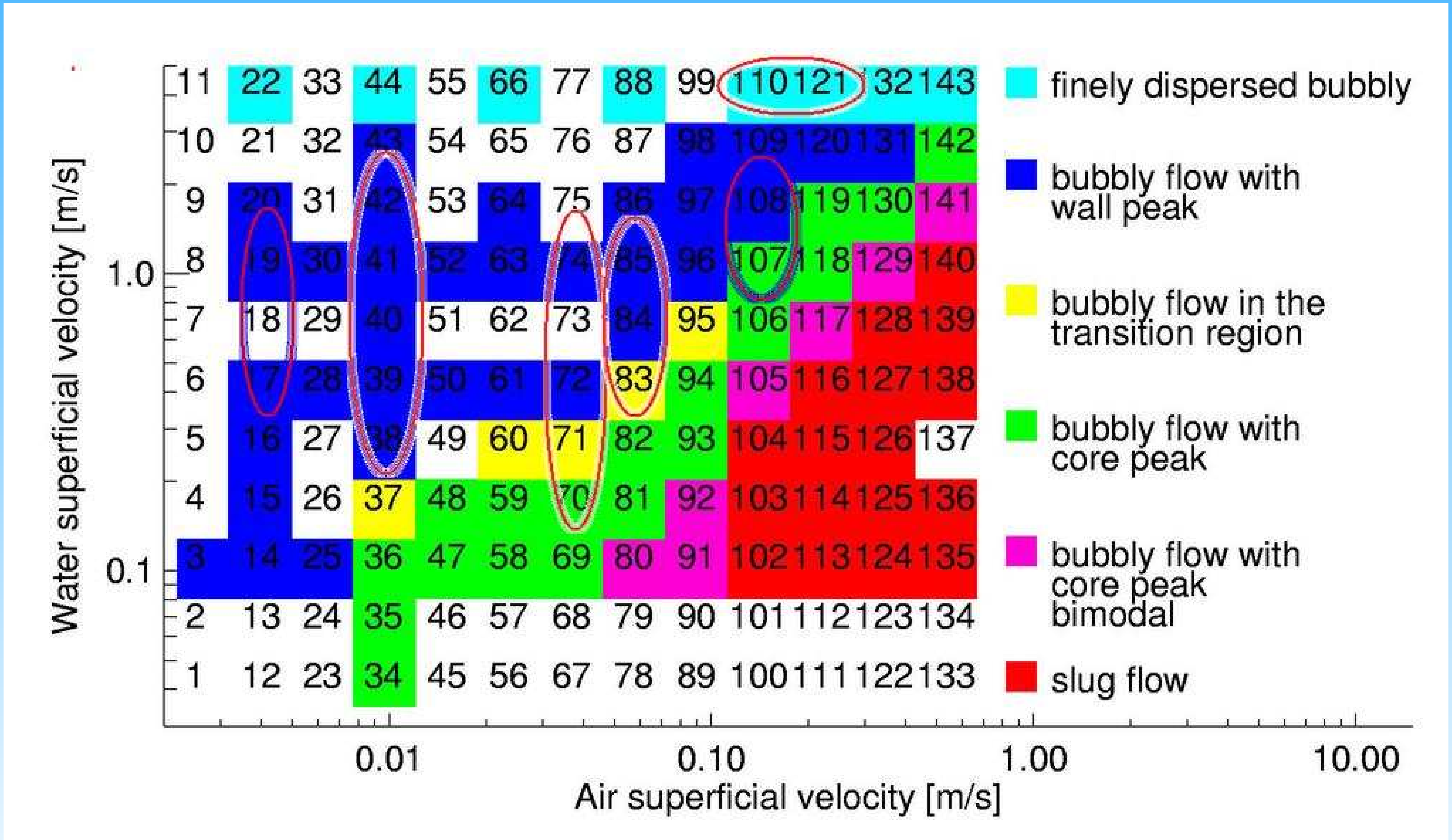
Wire mesh sensor measurements

Test section from gas injection: $L = 0.03$ to 3.03 m

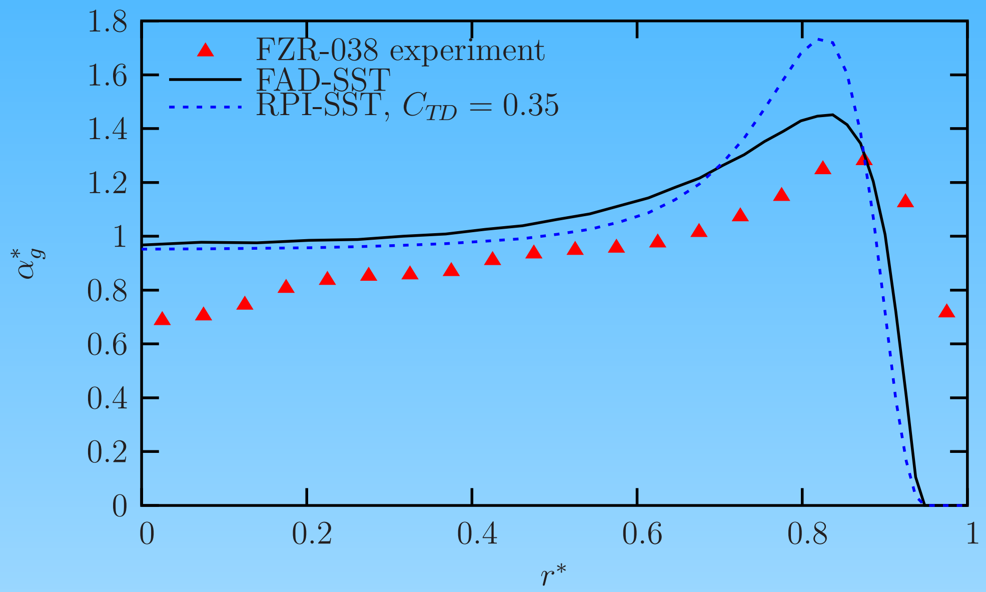
Injection nozzle arrangement:

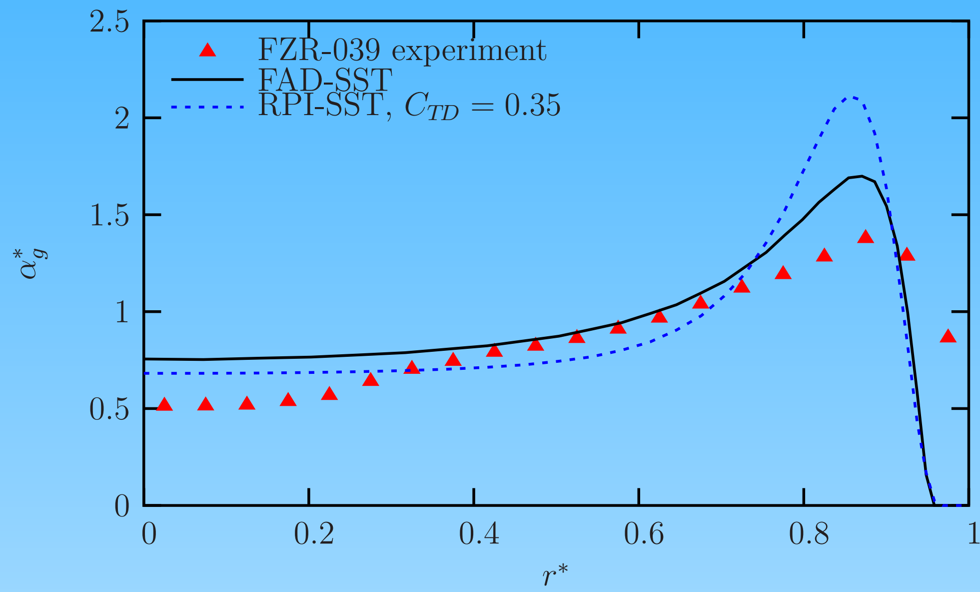


Test case definition



Two-fluid model Evaluation





For details refer to

J.-M. Shi, T. Frank, E. Krepper, D. Lucas, U. Rohde, H.-M. Prasser, ICMF2004, Paper No.400

Th. Frank, J.-M. Shi, and A. Burns, 3rd International Symposium on Two-Phase Flow Modeling and Experimentation, Pisa, Italy, 22-24 September, 2004

Poly-dispersed model Evaluation

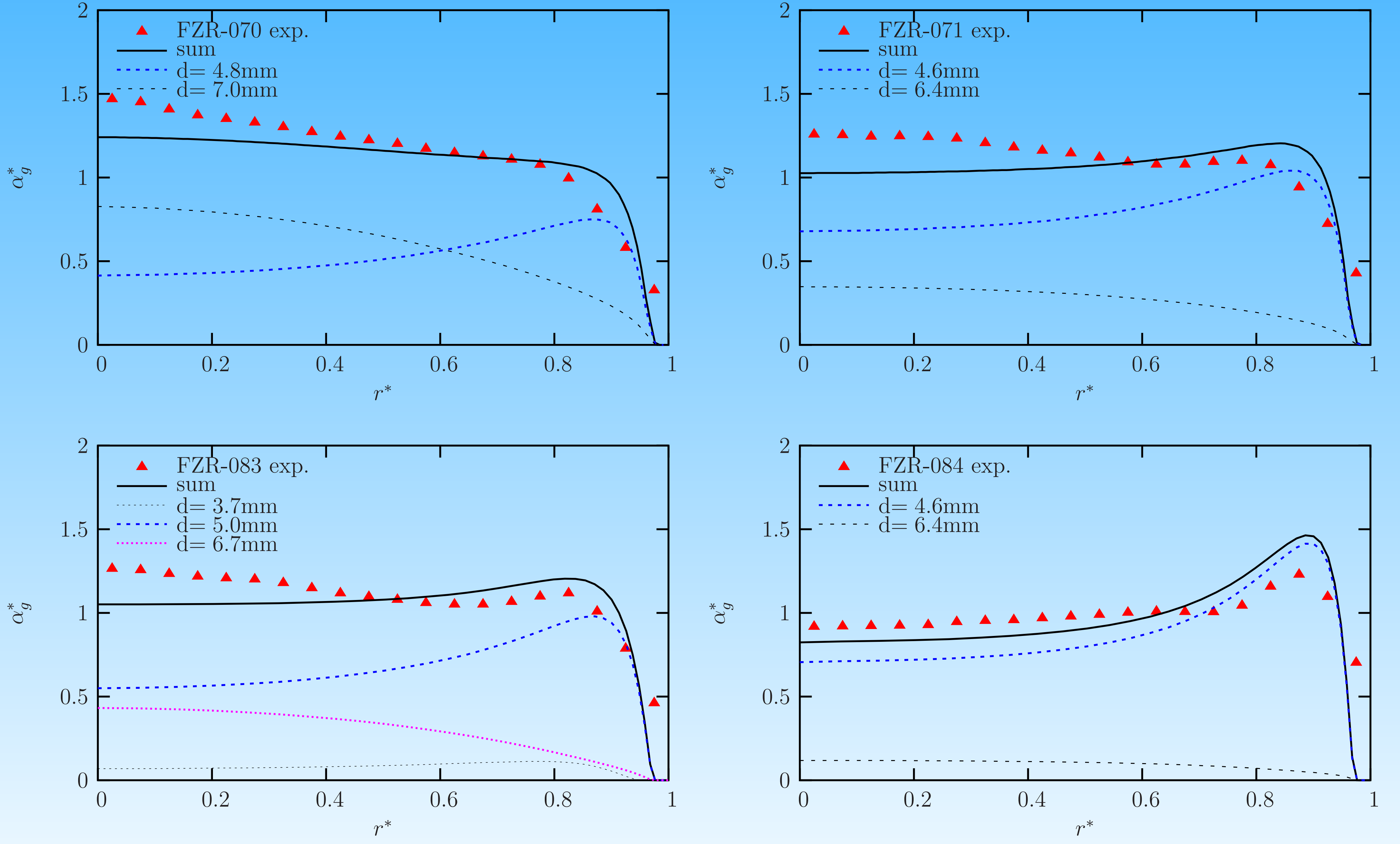
- Stationary, axisym., bubbly flow at the upper test section ($L/D = 59.2$)

- Data from measurements: superficial velocities, mean bubble diameter, local gas volume fraction

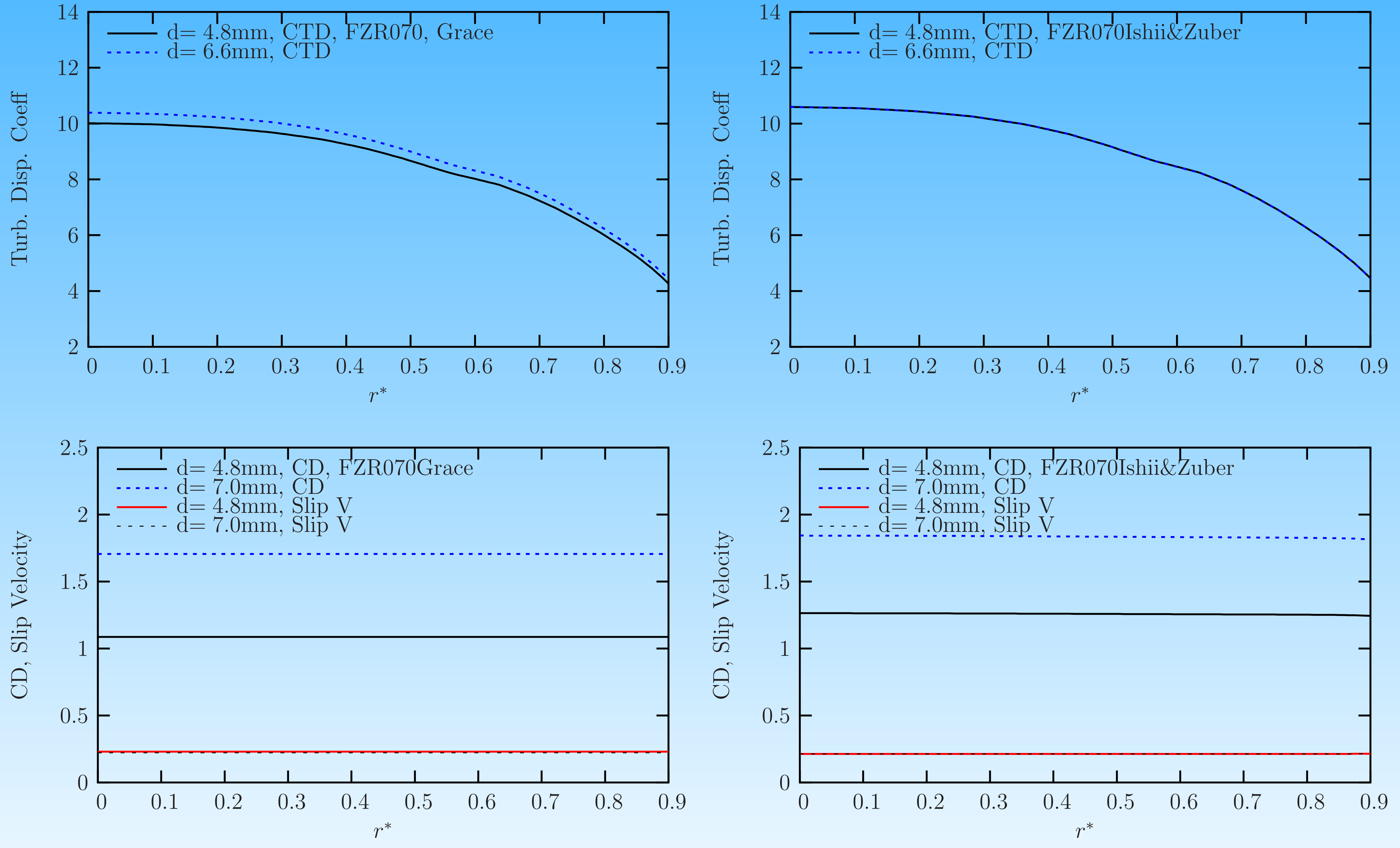
Index	Ul	Ug	Air	Air 1		Air 2		Air 3	
	[m/s]	[m/s]	VF[%]	dp	VF[%]	dp	VF[%]	dp	VF[%]
070	0.161	0.0368	22.86	4.8	12.20	7.0	10.66		
071	0.255	0.0368	14.43	4.8	11.27	6.6	3.16		
072	0.405	0.0368	9.09	4.6	8.18	6.4	0.91		
072	0.405	0.0368	9.09	3.8	2.48	5.0	5.70	6.4	0.91
083	0.405	0.0574	12.76	4.8	9.86	6.7	2.90		
083	0.405	0.0574	12.76	3.7	1.00	5.0	8.86	6.7	2.90
084	0.641	0.0574	8.95	4.6	8.24	6.4	0.71		
107	1.017	0.140	13.77	5.1	9.47	6.8	4.30		
108	1.611	0.140	8.69	4.7	8.10	6.3	0.59		
110	4.047	0.140	3.46	2.7	3.46				
110	4.047	0.140	3.46	2.4	1.71	3.4	1.75		

Ul, Ug–superficial velocity, dp–diameter [mm], VF–gas volume fraction.

Results from poly-dispersed model



TD coefficient, FZR070, $C_{TD} = \frac{3}{4}\rho_f \frac{\overline{C_D}}{d_p}\frac{\nu_{f,t}}{\sigma_f}\overline{|\mathbf{U_p} - \mathbf{U_f}|}$



TD coefficient, FZR110, $C_{TD} = \frac{3}{4}\rho_f \frac{\overline{C_D}}{d_p} \frac{\nu_{f,t}}{\sigma_f} \overline{|\mathbf{U_p} - \mathbf{U_f}|}$