DEVELOPMENT OF A MULTIPLE VELOCITY MULTIPLE SIZE GROUP MODEL FOR POLY-DISPERSED MULTIPHASE FLOWS

Jun-Mei Shi, Phil Zwart[1], Thomas Frank[2], Ulrich Rohde, and Horst-Michael Prasser

1. Introduction

Poly-dispersed multiphase flows involving particle allegation and breakage processes are widely encountered in engineering and industrial facilities. Design of these facilities and development of optimal processing techniques require a CFD tool for predicting the local particle number density and the size distribution. These quantities not only have a significant effect on the rate of mixing, reaction and the interfacial heat and mass transfer, but also a direct relevance to the hydrodynamics of the total system, such as the flow pattern and flow regime. The Multiple Size Group (MUSIG) model [1] available in the commercial codes CFX-4 and CFX-5 was developed for this purpose. Mathematically, this model is based on the population balance method and the two-fluid modeling approach. The dispersed phase is divided into N size classes. For each class a continuity equation taking into account of the inter-class mass transfer resulting from particle coalescence and breakup is derived from the population balance equation. By assuming all size groups to share the same velocity field, only one set momentum equations need to be solved for the entire dispersed phase. This assumption significantly reduces the computational cost. As a result, this model allows to use a sufficient number of particle size groups required for the coalescence and breakup calculation and has found a number of successful applications to large-scale industrial multiphase flow problems. Nevertheless, the assumption also restricts its applicability to homogeneous dispersed flows — the slip velocity of particles are approximately independent of particle size; and the particle relaxation time is sufficiently small relative to inertial time scales so that the asymptotic slip velocity may be considered to be attained almost instantaneously —. Hence we refer to the current implementation of the CFX MUSIG model as the homogeneous model.

Failures were reported in flows where this assumption ceases to be valid. One example is the bubbly flow in vertical pipes where the non-drag forces play an essential role on the bubble motion. Especially, the lift force acting on large deformed bubbles is mainly caused by the asymmetrical wake, which is in the opposite direction to the shear induced lift force on a small bubble [2,3]. For this reason, a core peak is measured for the volume fraction of large bubbles and a wall peak for those smaller [4,5]. Nevertheless, the CFX-4 MUSIG model failed to predict this bubble separation as reported in [6].

In general, the motion of particles of different sizes can be dominated by different forces or physical processes and the ratio of the particle response time to the flow convection time scale can cover a wide range. The non-drag forces on a particle are usually size-dependent. For example, we found that the turbulent dispersion of bubbles strongly depends on the particle size [6]. These inhomogeneous features in the dispersed phase are neglected in the homogeneous model. In order to take the inhomogeneous motion of different size classes into account, a full multiphase model, which treats each size group as a different dispersed phase with its own velocity field, has been proposed [8,9]. Nevertheless, this model requires to solve a complete set of transport equations of all dependent variables for each size group.

[1] ANSYS Ltd. Canada, Waterloo On.
[2] ANSYS Germany, Otterfing

Consequently, the required computational effort is as much as $N + 1$ times of a corresponding single phase flow problem. This disadvantage prevents its application to many practical multiphase flows.

In this report we summarize our efforts in developing an efficient inhomogeneous MUSIG model in cooperation with ANSYS CFX. A novel multiple velocity multiple size group model, which incorporates the population balance equation into the multi-fluid modeling framework, was proposed [10]. The original concept was to model bubbles with opposite lift force separately by using two velocity groups and allow each velocity group to have a sub-division of size classes. In this way, the inhomogeneous motion of the dispersed phase is considered efficiently along the line of the MUSIG model. This concept leads to a general framework covering all possible class model variants [11], namely dividing the dispersed phase into N fields (dispersed fluids), each allowing an arbitrary number of sub-size classes (e.g., M_i). We refer to it as the $N \times M$ model. In this sense the full multiphase model becomes the $N \times 1$ special case. The $N \times M$ model is still under evaluation and will be released in CFX5.8. Here we present the model concept and some results from pre-investigation based on the $N \times 1$ variant.

2. The $N \times M$ MUSIG model

Using the multi-fluid modeling approach we might model the dispersed phase by N fields (separated phases) according to the particle sizes to account for the inhomogeneity in the dispersed phase flow. Hence N velocity fields are to be solved for the dispersed phase. For this reason, we refer to these fields as velocity groups. We further divide each velocity group into a sub-division of size cuts (e.g., M_i) and assume that they share the same velocity field corresponding to this velocity group as in the homogeneous MUSIG model. Then only a continuity equation based on the population balance method has to be solved for the mass conservation of a sub-size class coupled with the coalescence and breakup processes. Without loss of the generality, the model equations are presented for an isothermal, laminar multiphase flow of Newtonian fluids and without mass transfer between the continuous and the dispersed phase.

The governing equations describing the mass and momentum conservation for the continuous phase are as follows:

$$\frac{\partial}{\partial t}(r_\ell \rho_\ell) + \nabla \cdot (r_\ell \rho_\ell \mathbf{U}_\ell) = 0 \qquad (1)$$

$$\frac{\partial}{\partial t}(r_\ell\, \rho_\ell \mathbf{U}_\ell) + \nabla \cdot (r_\ell\, \rho_\ell \mathbf{U}_\ell \mathbf{U}_\ell) = -r_\ell\, \nabla p - \nabla \cdot (r_\ell\, \underline{\underline{\mathcal{T}_\ell}}) + r_\ell\, \rho_\ell \mathbf{g} + \mathbf{F}_\ell + \mathbf{I}_\ell \qquad (2)$$

where r_ℓ is the volume fraction of the continuous phase, $\mathbf{F}_\ell$ the body force excluding the gravity and $\mathbf{I}_\ell$ the momentum interaction between the continuous and dispersed phase. $\underline{\underline{\mathcal{T}_\ell}}$ is the stress tensor defined as

$$\underline{\underline{\mathcal{T}_\ell}} = -\mu_\ell \left(\nabla \mathbf{U}_\ell + \nabla^{\mathbf{T}} \mathbf{U}_\ell) - \frac{1}{3}(\nabla \cdot \mathbf{U}_\ell)\, \underline{\underline{\mathbf{I}}} \right) \qquad (3)$$

Defining r_m to be the volume fraction of the velocity group m of the dispersed phase, its continuity equation can be written as

$$\frac{\partial}{\partial t}(\rho_m\, r_m) + \nabla \cdot (\rho_m\, r_m \mathbf{U_m}) = S_m \qquad (4)$$

where S_m is the mass source term, to be specified in eq. (9). The momentum equation can be expressed as

$$\frac{\partial}{\partial t}(\rho_m r_m \mathbf{U_m}) + \nabla \cdot (\rho_m r_m \mathbf{U_m} \mathbf{U_m}) = - r_m \nabla p - \nabla \cdot (r_m \underline{\underline{\mathcal{T}_\mathbf{m}}}) + r_m \rho_m \mathbf{g} + \mathbf{F_m} + \mathbf{I}_{\ell,\mathbf{m}} + \mathbf{I}_{\mathbf{d},\mathbf{m}} \tag{5}$$

where $\mathbf{I}_{\ell,\, m}$ represents the interaction with the continuous phase (e.g., the interfacial forces). The quantity $\mathbf{I}_{d,\, m}$ is introduced to denote the secondary momentum transfer, which is related to the mass transfer between the velocity groups resulting from particle coalescence and breakup. Specification of this term is described in detail in [10]. For simplicity the same pressure field as the continuous phase is assumed above.

The population balance equation is applied to each sub-size group. Assume that the size group i is a sub-division of the velocity group m. Defining r_d and r_i to be the volume fraction of the total dispersed phase and of the size group i, respectively, and f_i and $f_{m,i}$ to be respectively the size fraction of the size group i in the total dispersed phase and in the velocity group m, the different flavors of size fractions are related by

$$r_i = r_d f_i = r_m f_{m,i} \tag{6}$$

With these definitions, and recognizing that velocity fields are homogeneous for all size groups within a velocity group m, the population balance equation for the size group i leads to

$$\frac{\partial}{\partial t}(\rho_m r_m f_{m,i}) + \nabla \cdot (\rho_m r_m \mathbf{U}_m f_{m,i}) = S_{m,i} \tag{7}$$

where $S_{m,i}$ is the mass source term.

Then the mass source S_m in eq. (4) can be specified, i.e.

$$S_m = \sum_{i=1}^{N_m} S_{m,i} \tag{8}$$

where N_m is the number of the sub-divisions in the velocity group m, and obviously, we have $\Sigma_m S_m = 0$.

In the case that coalescence and breakup are the only mass transfer mechanism, $S_{m,i}$ can be expressed as follows

$$S_{m,i} = B_{i,B} - D_{i,B} + B_{i,C} - D_{i,C} \tag{9}$$

where $B_{i,B}$ and $D_{i,B}$ are respectively the birth and death rate of the size group i due to breakup and $B_{i,C}$ and $D_{i,C}$ are the coalescence-related counterparts. They are defined as follows

$$B_{i,B} = \rho_d r_d \sum_{j>i} B_{ji} f_j, \tag{10}$$

$$D_{i,B} = \rho_d r_d f_i \sum_{k<i} B_{ik}, \tag{11}$$

$$B_{i,C} = (\rho_d r_d)^2 \frac{1}{2} \sum_{j\le i} \sum_{k\le i} C_{jk} f_j f_k \frac{m_j + m_k}{m_j m_k} X_{jk\to i}, \tag{12}$$

$$D_{i,C} = (\rho_d r_d)^2 \sum_{j} C_{ij} f_i f_j \frac{1}{m_j} \tag{13}$$

where B_{ji} is the specific breakup rate from size group j to i, C_{jk} is the specific coalescence rate between size group j and k, m_i represents the mass of a single particle of the group i. $X_{jk\to i}$ is a factor projecting the corresponding part of the birth particle into the ith size group, defined as follows

$$X_{jk\to i} = \begin{cases} \frac{(m_j+m_k)-m_{i-1}}{m_i-m_{i-1}} & \text{if} \quad m_{i-1} < m_j + m_k \le m_i \\ \frac{m_{i+1}-(m_j+m_k)}{m_{i+1}-m_i} & \text{if} \quad m_i < m_j + m_k < m_{i+1} \\ 1 & \text{if} \quad m_j + m_k \ge m_{max} = m_i \\ 0 & \text{else} \end{cases} \tag{14}$$

with $\Sigma_i X_{jk\to i} = 1$. In addition, the sum of the net mass source over all size groups should vanish, i.e. $\Sigma_i (B_{i,B} - D_{i,B}) = 0$ and $\Sigma_i (B_{i,C} - D_{i,C}) = 0$.

3. Results based on the N × 1 variant

The $N \times M$ MUSIG model is to be released as a CFX5.8 new feature after the validation and evaluation. As a pre-investigation, the $N \times 1$ variant was implemented and applied to gas-liquid flows in a vertical pipe with an inner diameter D=195.3 mm, a test section of the TOPFLOW facility at FZR [12]. Detailed results were presented in [13]. As an example, the numerical results for the radial distribution of the bubble volume fraction α_g of both the total gas phase (r_d) and each velocity group (r_m) at various distances from the injection plane are displayed in Fig. 1 for the test case 074 corresponding to a superficial velocity of 0.0368 m/s for air and 1.017 m/s for water. A number of 4 velocity groups was applied in the simulations and the gas was assumed to be incompressible. The experimental data measured using the wire-mesh sensor technique [12] were also plotted for comparison. The gas was injected from 144 nozzles of 1 mm in diameter in experiments. In simulation the gas injection from each nozzle was approximated by a point mass source. The results confirm that the current model is capable of predicting the separation of bubbles of different size classes and the development of the radial distribution of the gas volume fraction along the pipe. Relatively larger deviations are observed between the simulation and measurements in the cases of smaller distances between the measurement plane and the gas injection. This is mainly due to the approximation introduced for the gas injection condition. In addition, the non-drag force models applied here have only been validated for fully developed flows rather than the developing flows close to the injection plane. Also, different from the simulation, the wire-mesh sensor was fixed in the experiments and measurement data were obtained by varying the gas injection planes. This causes offsets in the hydrostatic pressure between the corresponding positions in measurements and in simulation. Further investigation is to be carried out to evaluate the effects of these causes. Besides, the investigations also indicate a need in validation of the coalescence and breakup models.

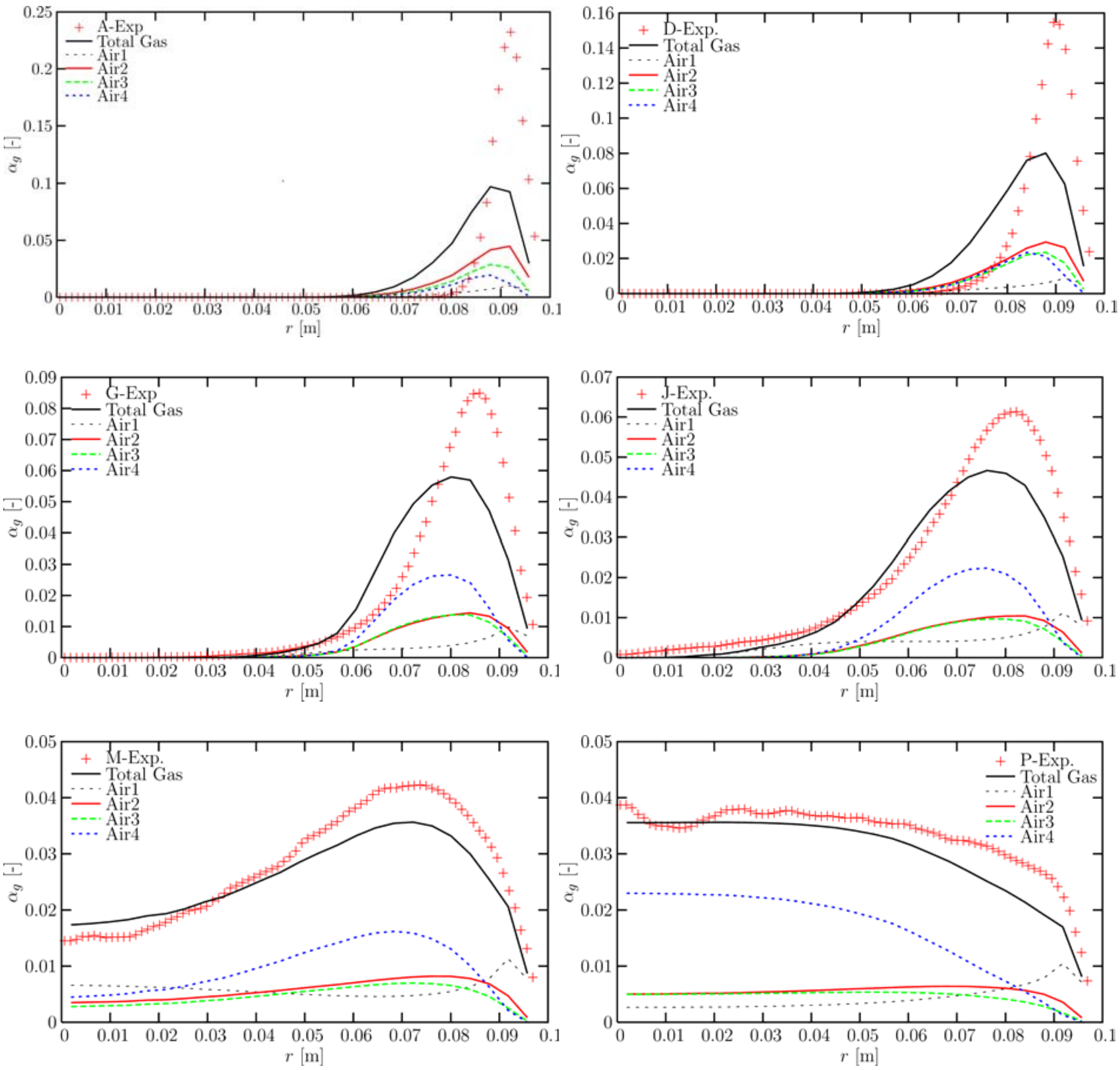

Fig. 1: Experimental and numerical results for the radial distribution of the gas volume fraction in a vertical pipe for the test case 074. The measurement planes denoted by A, D, G, J, M and P correspond to a distance 0.221, 0.494, 1.438, 2.481, 4.417 and 7.688 m, respectively, away from the gas injection.

4. Summary

A generalized multiple velocity multiple size group ($N \times M$ MUSIG) model applicable to inhomogeneous poly-dispersed multiphase flows is developed in cooperation with ANSYS CFX. The preliminary investigations using the $N \times 1$ variant show that this model is able to predict separation of bubbles of different sizes and the development of the radial distribution of the gas volume fraction along the pipe. The results also suggest a need in validation of the non-drag force models for developing flows and the coalscence and breakup model.

References

[1] S. Lo. Application of population balance to cfd modelling of bubbly flow via the MUSIG model. Technical Report AEAT-1096, AEA Technology plc, 1996.

[2] A. Tomiyama. Struggle with computational bubble dynamics. In Third International Conference on Multiphase Flow, ICMF 98, Lyon, France, 1998.

[3] A. Tomiyama, A. Sou, I. Zun, N. Kanami, and T. Sakaguchi. Effect of Eötvös number and dimensionless liquid volumetric flux on lateral montion of a bubble in a liminar duct flow. In A. Serizawa, T. Fukano, and J. Bataille, editors, Advances in Multiphase Flow, pages 3–15. Elsevier Science, 1995.

[4] H.-M. Prasser, E. Krepper, and D. Lucas. Evolution of the two-phase flow in a vertical tube - decomposition of gas fraction profiles according to bubble size classes using wire-mesh sensors. Int. J. of Thermal Sciences, 41:17–28, 2002.

[5] D. Lucas, E. Krepper, and H.-M. Prasser. Evolution of flow patterns, gas fraction profiles and bubble size distributions in gas-liquid flows in vertical tubes. Transactions of the Institute of Fluid-Flow Machinery, 112:37–46, 2003.

[6] E. Krepper and H.-M. Prasser. Measurements and CFX simulations of a bubbly flow in a vertical pipe. In AMIFESF Workshop, Computing methods for two-phase flow, pages 1–8. 2000.

[7] J.-M. Shi. Evaluation of turbulent dispersion of bubbles using a poly-dispersed model. Presentation at the 2004 Meeting of German CFD Association in Nuclear Safety Research, Rossendorf, May 05, 2004.

[8] P.M. Carrica, D. Drew, F. Bonetto, and R.T. Lahey Jr. A polydisperse model for bubbly two-phase flow around a surface ship. Int. J. Multiphase Flow, 25:257–305, 1999.

[9] A. Tomiyama and N. Shimada. A numerical simulation of bubble columns using a 3D multi-fluid model. In Proceedings of 3rd International Conference on Multiphase Flow (CD-ROM), Lyon, France, 1998.

[10] J.-M. Shi, E. Krepper, D. Lucas, and U. Rohde. Some concepts for improving the MUSIG model. Technical report, Institute of Safety Research, Forschungszentrum Rossendorf, March, 2003. submitted to CFX Germany.

[11] P. Zwart, A. Burns, and C. Montavon. Multiple size group models. Technical report, AEA Technology plc, Novermber, 2003. CFX-5.7.

[12] H.-M. Prasser, M. Beyer, A. Böttger, H. Carl, D. Lucas, A. Schaffrath, P. Schütz, F.-P. Weiss, J. Zschau. Influence of the Pipe Diameter on the Structure of the Gas-Liquid Interface in a Vertical Two-Phase Pipe Flow. 10^{th} International Topical Meeting on Nuclear Reactor Thermal Hydraulics (NURETH-10), Seoul, Korea, October 5-9, 2003; Conference Proceedings CD: A00308.

[13] J.-M. Shi, T. Frank, H.-M. Prasser, and U. Rohde. $N \times 1$ MUSIG model — implementation and application to gas-liquid flows in a vertical pipe. In ANSYS CFX & ICEM CFD Conference 2004, Dresden, 10.-12. November, 2004.

Acknowledgments

The model was proposed based on the extensive investigations carried out in Forschungszentrum Rossendorf. The financial support from Ministry of Economy and Labor (BMWA) of Germany to the project "TOPFLOW - Transient two phase flow test facility for generic investigation of two phase flows and further development and validation of CFD codes" is gratefully acknowledged. Thank Alan Burns (CFX Europe, Harwell) for valuable discussions and for providing useful documents.