A Multiple Field Multiple Size Group Model for Poly-Dispersed Gas-Liquid Flows – Part 1. Model Concepts and Equations

3rd Joint CFX & FZR Workshop on Multiphase Flows: Simulation, Experiments and Application

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31 May - 03 June, 2005, Dresden, Germany

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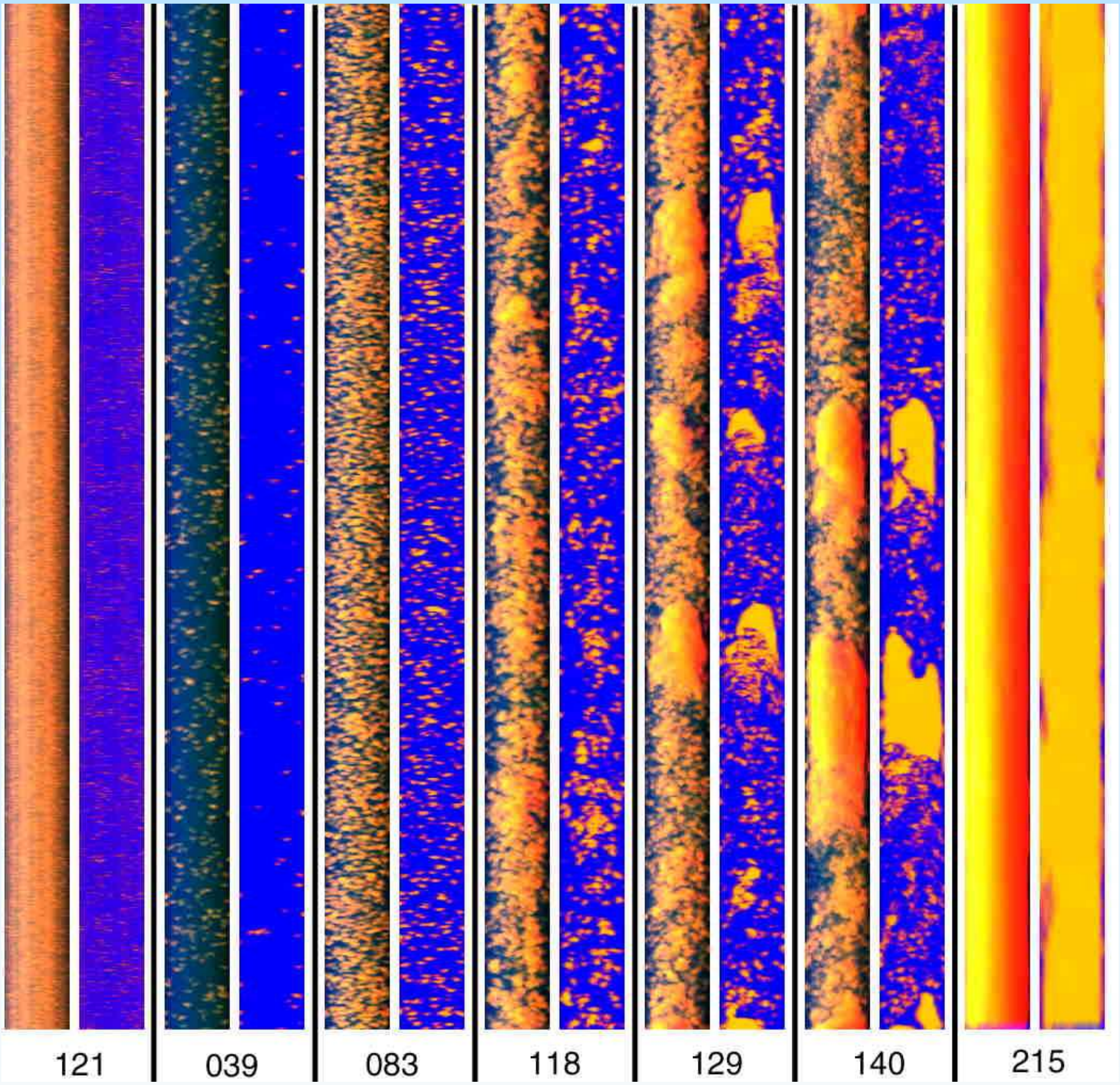
Outline of contents

- Background and Motivation
- Multi-fluid and population balance modeling
- Models in the literature
- A multiple field multiple size group model

Complex phenomena in gas-liquid flows



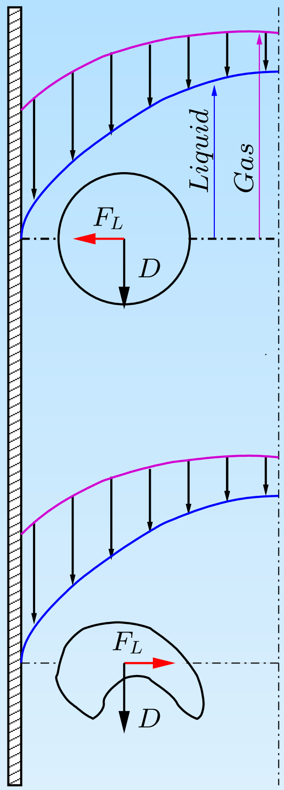
Flow Regimes

- Finely dispersed flow (121)
- Bubbly flow
  - Wall void maximum (039)
  - Transition region (083)
  - Core void maximum (118)
  - bimodal maximum (129)
- Slug flow (140)
- Annular flow (215)

Features

- Multiple morphology and length scales
- Inhomogeneous motions
- Breakup and coalescence
- Flow regime transition

Modeling industrial poly-dispersed flows



- Prefer Eulerian approach: high concentration, large scale
- Need multi-fluid models for inhomogeneous motion of particles:
  - diverse interfacial interaction depending on d_p
  - multiple length, time, and velocity scales,
    $\tau_p = \frac{4}{3}\frac{\rho_p}{\rho_f}\frac{d_p}{C_D\,|\mathbf{U_p}-\mathbf{U_f}|}$, $|\mathbf{U_p}-\mathbf{U_f}|$
- Bubble size distribution is a major operating parameter of the system hydrodynamics, e.g., flow pattern, transport and mixing.
- The breakup and coalescence model is important for predicting the bubble size distribution, flow development and regime transition.
- The population balance method is a suitable tool for this purpose.

- Motivation: to develop an efficient multi-fluid based population balance model for industrial poly-dispersed flow simulation.

Multi-fluid modeling (1)

Phase indicator function

$$X_k(\mathbf{x},t;i) = \begin{cases} 1 & \text{if } \mathbf{x} \text{ occupied by phase } k \text{ in realization } i \\ 0 & \text{otherwise} \end{cases}$$

Averaging operators

ensemble average $\bar{f}(\mathbf{x},t) = \int_{\mathcal{E}} f(\mathbf{x},t;\mu)\,\mathrm{d}\,m(\mu) = \lim_{N\to\infty} \frac{\sum_{i=1}^{N} f(\mathbf{x},t;i)}{N}$

phase-weighted average $\bar{f}_k(\mathbf{x},t) = \frac{\overline{f\,X_k}}{\overline{X_k}} = \lim_{N\to\infty} \frac{\sum_{i=1}^{N} f(\mathbf{x},t;i) X_k(\mathbf{x},t;i)}{\sum_{i=1}^{N} X_k(\mathbf{x},t;i)}$

Averaged variables

"volume fraction" $r_k = \overline{X_k}$

phase-weighted density $\rho_k = \frac{\overline{\rho\,X_k}}{r_k}$

Favré-averaged transport variables $\phi_k = \frac{\overline{\rho\,X_k\,\phi}}{\overline{\rho\,X_k}} = \frac{\overline{\rho\,X_k\,\phi}}{\rho_k\,r_k}$

Multi-fluid modeling (2)

- Governing equations established from averaging, $\overline{X_k \text{ (instant eqs.)}}$:

$$\frac{\partial}{\partial t}(\rho_k r_k) + \nabla \cdot (\rho_k r_k \mathbf{U}_k) = S_{rk} \ , \qquad \sum_{k=1}^{N} r_k = 1$$

$$\frac{\partial}{\partial t}(\rho_k r_k \mathbf{U}_k) + \nabla \cdot (\rho_k r_k \mathbf{U}_k \mathbf{U}_k) = -r_k \nabla P - \nabla \cdot (r_k \mathbf{\Pi^k}) + \mathbf{F_k} + \mathbf{I_k} + S_{\mathbf{U}k}$$

- Need closure models for interfacial momentum transfer:

$$\mathbf{I_k} = \underbrace{\mathbf{F_D}}_{drag\ force} + \underbrace{\mathbf{F_L}}_{lift\ force} + \underbrace{\mathbf{F_W}}_{wall\ force} + \underbrace{\mathbf{F_{VM}}}_{virtual\ mass} + \underbrace{\mathbf{F_{TD}}}_{turbulent\ dispersion}$$

- Population balance model for coalescence and breakup:
  - discretisation of the dispersed phase into N_S size groups, $r_{d,i}$ $(i = 1..N_S)$

$$\frac{\partial}{\partial t}(\rho_d r_{d,i}) + \nabla \cdot (\rho_d r_{d,i} \mathbf{U}_i) = B_{B,i} - D_{B,i} + B_{C,i} - D_{C,i}$$

Models available in the literature (1)

The $N+1$ or $N \times 1$ model

- The full multi-fluid model:

Phase	Variables				
Continuous phase	$\mathbf{r}_\ell$,	$\mathbf{U}_\ell$,	$\mathbf{V}_\ell$,	$\mathbf{W}_\ell$,	$\mathbf{P}$
Dispersed phase size group i	$\mathbf{r_{d,i}}$,	$\mathbf{U}_{d,i}$,	$\mathbf{V}_{d,i}$,	$\mathbf{W}_{d,i}$,	$(i = 1..N_S)$

- Constraint equation: $\mathbf{r}_\ell + \sum_i^N \mathbf{r_{d,i}} = 1$
- Taking the full flow inhomogeneity into account
- Solving $4 \times (N_S + 1) + 1$ eqs. (laminar case), computationally expensive.
- refer to

  Carrica et al., *Int J. Multiphase Flow 25:257, 1999*;

  Tomiyama& Shimada, *J. Pressure Vessel Tech, 123:510, 2001*.

Models available in the literature (2)

The CFX homogeneous MUSIG model (Lo, 1996)

- The two-fluid model: one velocity field for the dispersed phase

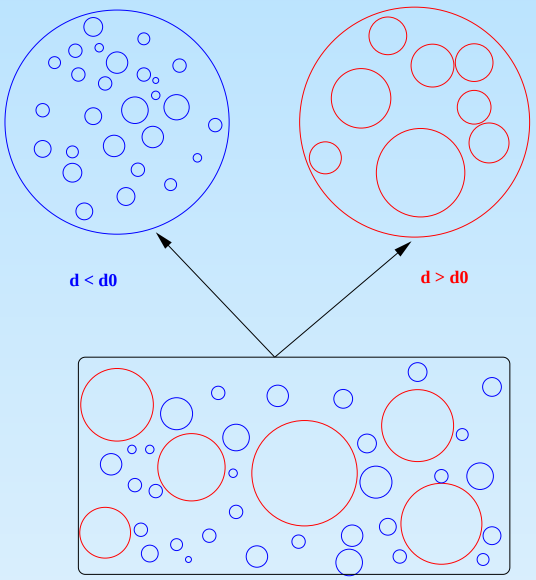
Phase	Variables
Continuous phase	$\mathbf{r}_\ell$, $\mathbf{U}_\ell$, $\mathbf{V}_\ell$, $\mathbf{W}_\ell$, $\mathbf{P}$
Dispersed phase	$\mathbf{r_d}$, $\mathbf{U_d}$, $\mathbf{V_d}$, $\mathbf{W_d}$
Size group i	$\mathbf{r_{d,i}}$ $(i = 1..N)$

- Constraint equation: $\mathbf{r}_\ell + \mathbf{r_d} = 1$ where $\mathbf{r_d} = \sum_i^N \mathbf{r_{d,i}}$
- Solving $N_S + 2 \times 4 + 1$ eqs., allowing a sufficient number of size classes.
- Applies to homogeneous poly-dispersed flows with weak size effect.
- Fails to handle flows with size-dependent inhomogeneities, e.g., *segregation of different size groups due to opposite interfacial forces, strongly size-dependent time and velocity scales.*

The multi-field multi-size group ($N \times M$ MUSIG) model

The two-velocity group MUSIG model (Shi et al., 2003)

- Dividing bubbles into 2-velocity groups based on the sign of the life force
- Further size discretisation in each velocity group
- Population balance modeling of mass transfer between all size groups
- Solving $N_S + 3 \times 4 + 1$ eqs., an efficient model



The N_V-velocity group extension (Zwart, Burns and Montavon, 2003)

- Using N_V-velocity groups according to bubble hydrodynamics, e.g., interfacial forces, transport velocity, particle response time
- solving $N_S + (N_V + 1) \times 4 + 1$ eqs., a generalized framework for all possible class models

$N \times M$ MUSIG model (2)

- Continuity equations for the velocity and size groups

$$\frac{\partial}{\partial t}(\rho_m r_m) + \nabla \cdot (\rho_m r_m \mathbf{U_m}) = S_m, \qquad m = 1 \,..\, N_V$$

$$\frac{\partial}{\partial t}\left(\rho_m r_m f_{m,i}\right) + \nabla \cdot \left(\rho_m r_m \mathbf{U_m} f_{m,i}\right) = S_{m,i}, \qquad i \in [N_m^0, N_m^1] \subset [1, N_S]$$

$$r_i = r_d f_i = r_m f_{m,i}, \qquad r_d = \sum_{m=1}^{N_V} r_m = \sum_{i=1}^{N_S} r_i, \qquad r_m = \sum_{i=N_m^0}^{N_m^1} r_i$$

$$r_\ell + r_d = 1, \qquad \sum_{i=1}^{N_S} f_i = 1, \qquad \sum_{i=N_m^0}^{N_m^1} f_{m,i} = 1$$

- Mass source terms due to breakup and coalescence

$$S_{m,i} = B_{i,B} - D_{i,B} + B_{i,C} - D_{i,C}, \qquad S_m = \sum_{i=N_m^0}^{N_m^1} S_{m,i}$$

$N \times M$ MUSIG model (3)

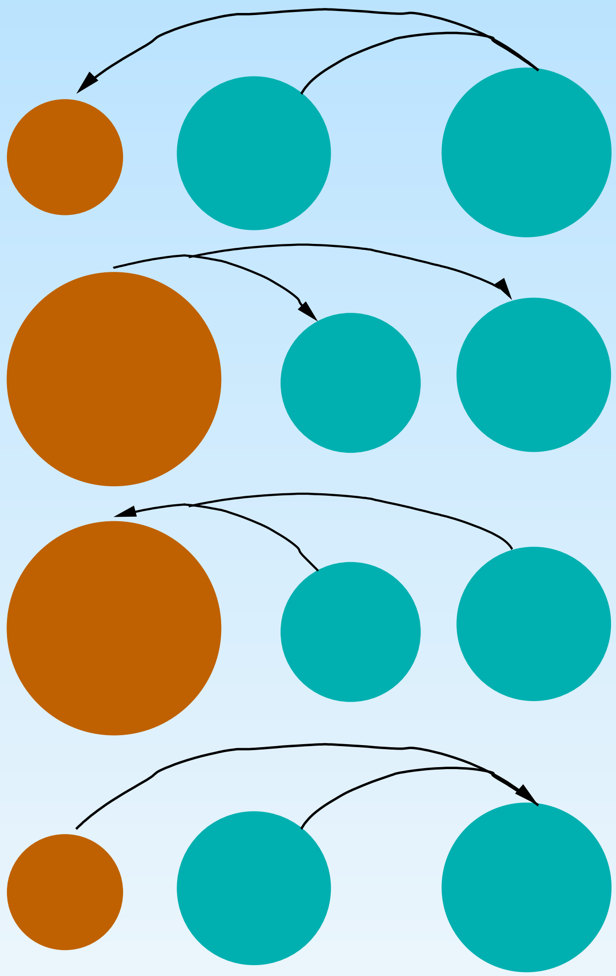
- Mass sources due to breakup and coalescence

$$B_{i,B} = \rho_d \, r_d \sum_{j>i} B_{ji} \, f_j$$

$$D_{i,B} = \rho_d \, r_d \, f_i \sum_{k<i} B_{ik}$$

$$B_{i,C} = (\rho_d \, r_d)^2 \, \frac{1}{2} \sum_{j \leq i} \sum_{k \leq i} C_{jk} \, f_j \, f_k \frac{m_j + m_k}{m_j \, m_k} \, X_{jk \rightarrow i}$$

$$D_{i,C} = (\rho_d \, r_d)^2 \sum_{j} C_{ij} \, f_i \, f_j \, \frac{1}{m_j}$$



$$\sum_{i=1}^{N_V} S_m = \sum_{i=1}^{N_S} S_{m,i} = 0, \qquad \sum_{i=1}^{N_S} (B_{i,B} - D_{i,B}) = 0, \qquad \sum_{i=1}^{N_S} (B_{i,C} - D_{i,D}) = 0$$

Implementation and model Evaluation

- The $N \times M$ MUSIG model has been implemented in ANSYS CFX10 (Phil Zwart, ANSYS Canada, Waterloo)
- Model evaluation based on measurement data will be presented by Thomas Frank of ANSYS Germany, Otterfing

Acknowlegment

Thank Ulrich Rohde of FZR and Alan Burns of ANSYS Europe and for valuable discussions